Advertisement.

THE *many Editions* Mr. Coggeshall's Art of Practical Measuring *has gone through, is a sufficient Proof of its Worth. The Corrections and Improvements that have been made by the Ingenious Mr.* Ham *has rendered it generally useful: But to make the Work still more so, it has been thought proper to add, a short* Treatise of Practical Geometry *and the* Principles of Plain Trigonometry, *with the Application thereof by a few Problems, which are here drawn up in a plain and easy manner for the Use of Learners.*

ERRATA.

PAGE 7. *line* 7. *for* A *read* As, *p.* 9. *l.* 20. *after* XB *add,* then cut off AC equal to AE, *l.* 25. *for Fig.* 6. *r. Fig.* 5. *p.* 12. *l.* 13. *for* through F, *r.* through Q. *p.* 18. *l.* 7. *for* DG *r.* GB *and for* HG *r.* AG, *p.* 28. *l.* 28. *Marg.* add, *Fig.* 4. *p.* 47. *l.* 4. *for* Sine *r.* Line, *p.* 55. *l.* 1. *add* SECT. IV. *l.* 5. *Marg.* add, *Fig.* 19. *l.* 25. *Marg.* add, *Fig.* 20. *p.* 56. *l.* 21. *Marg.* add, *Fig.* 21.

A

COMPENDIUM

OF

Practical Geometry

AND THE

PRINCIPLES

OF

Plain Trigonometry,

With the APPLICATION thereof.

By *G. THOMSON*, A.M.

LONDON:

inted for JOHN OSWALD, at the *Rose and Crown*, in the *Poultry*, near *Stocks-Market.*
MDCCXXXII.

Price One Shilling.

A COMPENDIUM OF Practical GEOMETRY.

DEFINITIONS.

A POINT is that which has no Parts.

2. A Line is Length, without Breadth, and is produced by the flowing of a Point.

3 Points are the Terms of a Line.

. A right Line, as A B, is that which lies *Fig.* 1.
nly betwixt its Terms.

. Parallels are ſuch right Lines in the ſame Plane, *Fig.* 2.
GN and CF, which, if infinitely produced on
h Sides, would never meet.

. A Surface has only Length and Breadth; whoſe
ms are Lines.

. A plain Surface lies evenly betwixt its Terms,
is produced by the Motion of a right Line.

. A Body is a Magnitude long, broad and deep;
ſe Terms are Surfaces. *A Body therefore has*

B *three*

three Dimensions, a Surface two, a Line one, and Point none.

Fig. 3. 9. A plain Angle is the Inclination of two Li AB and AC to one another in the same Pla which touch each other in the Point A, but so not to make one right Line.

10. The Lines forming an Angle are called Sides, or Legs.

11. The Vertex, or Top, of the Angle is wh the Legs meet, as in the Point A.

12. A right-lined Angle is made of streight Lin a curvilinear Angle of crooked Lines, as B; and mixed one of a streight Line and a crooked one, as

Fig. 3. 13. A single Angle is designed by one Letter at Top, as X; but when there are more at one Poi they are expressed by three Letters, BAF; wher that at the Top of the Angle is placed in the Midd

14. Angles are equal, if, when the Tops are one on another, the Sides of the one exactly fall up the Sides of the other; but it is not required Sides should be of the same Length.

Fig. 4. 15. When a right Line, AD, standing on anot right Line CB, makes on both Sides thereof the gles CAD, and DAB, equal between themse each of these Angles is called a right one; and right Line AD standing on the other, is calle Perpendicular Line.

16. An obtuse Angle, CAX, is greater tha right one.

17. An acute Angle, XAB, is less than a right

These two Angles are often called oblique Angles.

18. A Triangle is a plain Surface contained der three right Lines.

Fig. 5. 19. An equilateral Triangle has all its three S equal.

Fig. 6. 20. An Isoceles, or Equicrural Triangle has two Sides equal.

Fig. 7. 21. A scalene Triangle, has all its three Sides equal.

Fig. 8. 22. A right angled Triangle has one right An as B.

The Side AC, *opposite to the right Angle* B, *is*

l the Hypothenuse, the longest Side C B, *of the other*
wo, including the right Angle, is called the Base and
e third Side B A *the Perpendicular, or Cathetus.*

23. An obtuse angled Triangle has one obtuse An- *Fig.* 7.
e, as X.

24. An acute angled Triangle has all its three An- *Fig.* 5.
es acute, as A, B and C.

25. The Height of a plain Triangle, is the Length *Fig.* 6,
a Line A D, let fall perpendicular from any of its and 7.
gles upon the Side opposite to that Angle from
ence it falls, and it may either fall within, or
thout the Triangle.

26. A Circle is a plain Figure contained under *Fig.* 9.
e Line called the Circumference, or Periphery, to
ich all right Lines, drawn from a certain Point C
led the Center) are equal.

27. The Diameter is a right Line A B, drawn
ough the Center, and terminated on both Sides
the Circumference; and it divides the Circle
two equal Parts.

8. The Semidiameter, or Radius, is a right Line
, drawn from the Center to the Circumference.

9. A Quadrant, A E, is the Quarter of a Circle
e by the Radius E C, perpendicular to the Dia-
r A B, at the Center C, cutting the Periphery
e Middle at E.

. A Chord or Subtense, A G, of an Arch is a
t Line cutting the Circle into two unequal Parts,
is less than the Diameter.

. A Segment, of a Circle, is a Figure included
ixt the Chord and that Arch of the Periphery
ff by the Chord; and is either greater or less
a Semicircle, as A E G, or A F G.

. A Sector is a Part of a Circle contained under
Semidiameters C D, and C B, and an Arch B D,
h is the Measure of the Angle B C D, intercept-
twixt the Semidiameters.

l the Angles of Sectors are Angles at the Center.

. An Angle in the Segment of a Circle, F A G,
tained under the Lines F A and A G, drawn
one Point A of the Circumference to the Ends
e Segment, F and G.

34. An Angle of a Segment, as A, is containd under the right Line A F, and Part of the Ci cumference A F.

35. An Angle, G A B, is said to stand on the Ci cumference G B, as being opposite to it.

The Circumference is divided into 360 *Degrees, t Semicircumference into* 180, *and the Quadrant into* 9

Fig. 10. 36. A Rectangle, A B C E, or a right angled P rallelogram, has four right Angles, and consequent equi-angled and its opposite Sides equal.

Fig. 11. 37. A Square, A B C D, has equal Sides, and right angled, and consequently equi-angled.

Fig. 12. 38. A Rhombus, A B C D, is equilateral, but n equi-angled.

Fig. 13. 39. A Rhomboides, B Z, has its opposite Sides a Angles equal, but is neither equilateral, nor equi-a gled.

Fig. 12, and 13. 40 The Heighth of a Rhombus, or Rhomboid is a Line B E, or C F, let fall perpendicular fr any Angle upon the Side opposite to that Angle, a may be either within or without the Figures.

All other Figures that differ from these abovem tioned, are called Trapezia.

Fig. 10. 41. The Diameter, or Diagonal, of a Paralle gram, and every Quadrilateral Figure, is a ri Line E B, drawn through the opposite Angles, a divides the Figure into two Triangles.

Fig. 14. 42. The external Angle of a right Lined Figu arises without the Figure, when the Side is pro ced. Such are, A E B, B F C, C H E.

Every Figure therefore has so many external An as it has Sides, or internal Angles.

All other Figures, that have more than Four Si are called Polygons, whether regular, or irregular.

Polygons are named according to the Number of t Sides, as a Pentagon is that which has five Sides Hexagon six, an Heptagon seven, an Octogon eight,

Fig. 15. 43. A Parallelepiped is a Body bounded by six rallelograms, whereof the two opposite ones ar way. equal and parallel

Fig. 16 44. A Cylinder is a Body made by the Rotation right angled Parallelogram, A B C D, round one Sides, till it end where it began.

45. A Pyramid is a Body E F, bounded by ſeveral Triangles, E C A, E A F, E A D, E D B, making the Surface for the Baſe F, meet all in one Point E. *Fig.* 17.

46. A Cone is a Body made by the Rotation of a right angled Triangle, A B C round a Circle, the angular Point of the right Angle being fix'd in the Center. *Fig.* 18.

47. A Priſm is a Body having two parallel Planes ſimilar and equal; and the others Parallelograms, as AD. *Fig.* 19.

48. A Sphere is a Body produced by the Motion of a Semicircle B D E round its Diameter B D, which remains immoveable. *Fig.* 20.

49. A Tetrahedon, or Pyramid, is a Body contained under four equal and equilateral Triangles. *Fig.* 21.

50. An Hexahedron, or Cube, is a Body (like a Dice) bounded by ſix Squares. *Fig.* 22.

51. An Octahedron, is a Body bounded by eight equal and equilateral Triangles. *Fig.* 23.

52. A Dodecahedron, is a Solid contained under twelve equal and equilateral Pentagons. *Fig.* 24.

53. An Icoſahedron, is a Body contained under twenty equal and equilateral Triangles. *Fig.* 25.

Theſe laſt five, are called the five regular Bodies.

Geometrical Problems.

PROBLEM I.

To divide a given Line A B, *into two equal Parts.* *Fig.* 3.

FROM the Centers A and B, deſcribe two Circles, with the Radius more than half the Line, cutting one another in C and D, and ſo draw the right Line C D. The ſame biſects the given Line A B.

Hence a Perpendicular may be raiſed upon the Middle of a given Line, or one may be let fall from the Points C *and* D, *to the given Line.*

PROB. II.

From a given Point A, *in a given right Line* C B, *to raiſe a Perpendicular* A D. *Fig.* 4.

From the Point A, take the equal Lines A C,

 and

and A B, from the Centers C and B, describe two Circles cutting one another in D. The Line drawn from A to D, will be the Perpendicular.

PROB. III.

Fig. 26. *From a given Point* A, *without a right Line* L X, *to let fall a Perpendicular to that Line.*

From the Center A, describe a Circle, cutting the given Line L X in C I. Bisect the Line C I, with the Line A B. This A B, is the Perpendicular.

PROB. IV.

Fig. 27. *To raise a Perpendicular upon the End of a given Line* A B.

Upon any Point out of the given Line, as at C, describe a Circle passing through the Point B, from which the Perpendicular is to be raised. From the Point A, where the Circle cuts the given Line, draw the Diameter A C D. From the Point D, draw the Line D B, which will be the Perpendicular.

These three last Problems *are easily done by the Help of a Square.*

PROB. V.

Fig. 2. *Thro' a given Point* F, *to draw a Parallel to a given right Line* A B.

Take any Point in the given Line, as X, with the Distance X F, describe a Semicircle. Take the Arch C G equal to the Arch F N. Thro' the Points C and F, draw the Line C F, which will be the Parallel.

PROB. VI.

Fig. 28. *To divide an Angle*, B A C, *into two equal Parts.*

Take from the Sides of the Angle two equal Lines A E and A D; then from the Centers E and D, describe two equal Circles, cutting one another in F, and draw the Line F A, This bisects the Angle.

Hence, an Angle may be divided into 4, 8, 16. &c. *equal Angles,* viz. *by bisecting each Part again.*

PROB. VII.

Fig. 29. *At a given Point* B, *in a right Line, to make an Angle equal to a given one* A.

From the Point A, with any Radius, describe an Arch N M, and from the Point B, with the same Radius, describe the Arch F G. Take the Arch F X equal to the Arch N M. Then thro' the Points B and X, draw a right Line, the Angle F B X, will be equal to the given one A.

A Proportion is of great Use, I have given the following Sketch.

QUANTITIES are in Proportion to one another, when the first Term contains the second, as often as the third contains the fourth, or when the first is as often contained in the second as the third is in the fourth. So 12 : 6 : 4 : 2. And 3 : 9 : 4 : 12 are alike; because in the former Example 6 and 2, are contained twice in the respective Antecedents, and so the Proportion of the Antecedents is double to the Consequents. In the other Example the Proportions are also alike, because the Consequents 9 and 12, contain their respective Antecedents three times, and so the Proportion of the Antecedents to the Consequents is subtriple.

See, in short, all the different Ways how like Proportions may be changed and ordered among themselves, so that the emerging Proportion on both Sides may be still alike.

Let it be	16 : 8 : : 4 : 2
Then it will be by Alternating.	16 : 4 : : 8 : 2
Inverting	8 : 16 : : 2 : 4
Compounding	16+8 : 8 : : 4+2 : 2
Dividing	16−8 : 8 : : 4−2 : 2
Converting	16 : 16+8 : : 4 : 4+2
	16 : 16−8 : : 4 : 4−2
	16+8 : : 8 : : 4+2 : 2
Comparing orderly	16 : 8 : : 4 : 2 & 8 : 4 : : 2 : 1
Then	16 : 4 : : 4 : 1
Comparing disorderly	16 : 8 : 1 : 4 : 2 & 8 : 32 : : 1 : 4
Then	16 : 32 : : 1 : 2

A

A Geometrical Construction of Continual Proportion.

Fig. 52. THIS Scheme is composed of Isoceles Triangles. Let d x y be a given Triangle: on one of its longest Sides, d x, make an Isoceles Triangle x d n, which is similar to the given one, because the Angle, x d n, is common to both: Then on d n make the Triangle b d n, and on b d make the Triangle b d s, *&c.*

Therefore, as the longest Side of the one, is the shortest Side of the other, and they are all similar. The Proportion will be, as d y : d x : : d x : x n : : x y : n b : : n b : b s : : b s : s t : : s t : r t : : r t : r z, &c.

PROB. VIII.

Fig. 31. *To divide a Line* A B, *according to a given Proportion, as* F I *to* I L.

Draw an infinite Line A Z, from which take A Q, Q R equal to F I and I L, and from R draw R B; parallel to this draw Q C. The Thing is done.

PROB. IX.

Fig. 32. *To divide a Line* A B *in like manner, as another given one* A I *is divided in* F *and* C.

Let the Line I B join the Extremities of the two Lines, draw Parallels to this from the Points F and C, meeting the Line A B, to be cut in L and Q. The Thing is done.

PROB. X.

Fig. 33. *Two Lines* A B, *and* B C, *being given to find a third Proportional.*

Draw A C, and from B A produc'd, take A F equal to B C; through F draw the infinite Line F x parallel to A C: And let B C be produc'd, till it meet x L in L. A B is to B C, as B C to C L.

PROB. XI.

Fig. 34. *Three Lines* A B, B C, *and* A F, *being given to find a fourth Proportional.*

Make

Make at Pleaſure the Angle F A C, and take upon A C the Line A B and B C, draw the Line B F; to which draw C Z parallel, let A F produced to A L, meet C Z. A B is to B C, as A F is to F L.

PROB. XII.

Two Lines A C *and* C B, *being given to find a Mean Proportional.* Fig. 35.

Biſect the compound Line A B in O, and from the Center O, deſcribe a Semicircle; from C erect a perpendicular meeting the Circumference in F. A C is to C F, as C F is to C B.

Hence all Triangles may be turned into Squares, by finding a mean Proportional between the Baſe and Half the Perpendicular.

PROB. XIII.

To cut a right Line A B, *in extream and mean Proportion.* Fig. 36.

From A erect a Perpendicular A F equal to A B, biſect A F in X: draw X B; from F A produced, cut off X I equal to X B. The Thing is done. For A B is to A C, as A C to C B.

Hence all Squares, Parallelograms, and Triangles may be divided into extream and mean Proportion.

PROB. XIV.

Upon a given Line A B, *to make an equilateral Triangle.* Fig. 6.

From the Center A, with the Diſtance A B, deſcribe an Arch; and from the Center B, with the ſame Diſtance, deſcribe another Arch, cutting the former in C; from which Point draw the Lines C A and C B. The Triangle A B C, will be an equilateral one.

PROB. XV.

To make a Triangle of three given Lines A B, C G *and* D F, *whereof any two of them muſt be greater than the third.* Fig. 37.

Let D F, one of the given Lines be taken, and one of its Extremities for the Center, with the Diſtance of

of the other given Line C G, describe an Arch; from the other End of D F, with the Distance of the third Line A B, describe another Arch cutting the former in E; then draw the Lines D E and E F, and they will form the Triangle required.

Hence a Triangle may be made equal to any given one.

PROB. XVI.

Fig. 38. *To divide a Triangle* ABC, *into any Number of equal Parts,* e. g. *into five.*

First, Divide the longest Side B C into five equal Parts, join the fifth Part D to A; divide the longest of the other two Sides into four equal Parts, join the fourth Part G to D; divide the remaining Part D G, into three equal Parts; join F to G; divide the Remainder G C, into two equal Parts, and join E to F. The Triangle ABC is divided into five equal Triangles, *viz.* ABD, ADG, GDF, GFE, and EFC.

PROB. XVII.

Fig. 39. *To divide a Triangle* A B C, *into any Number of equal Parts, by right Lines drawn from a given Point* D *in one Side,* e. g. *into three.*

Divide the Side A B into three equal Parts in the Points E and F, and join C D; draw to it thro' the Points of Division E and F, the Parallels E G and F H, which will give in the Sides A C and B C, the Points G and H: thro' which, and the given Point D, draw the Lines D G and D H, which will divide the Triangle into three equal Parts.

If the given Point D be in the Middle of the Side A B, the Triangle can be divided into six equal Parts, by dividing each of the two Sides A C and B C into three equal Parts at the Points L, G, M and H, and joining the Lines D L, D G, D H, D M, *&c.*

PROB. XVIII.

Fig. 40. *To divide a Triangle into three equal Parts, by Lines perpendicular to the Side* B C.

Draw A D perpendicular to B C from its opposite Angle, and divide either of its Segments, *e. g.* B D into three equal Parts at E and F; take in B C a mean Proportional

Proportional B G, betwixt B C and B F of the Segment B D, and take B D a mean Proportional betwixt BC and ⅔ BE of the Segment B D; and draw from G and H upon B C the Perpendiculars G I and H K, which will divide the Triangle ABC into three equal Parts.

The Triangle ABC may, in like manner, be divided into equal Parts by right Lines, making an Angle B A D with the Side B C equal to the given one B, and having found, as before, G and H, draw, thro' these Points, the Lines G I and H K parallel to A D.

PROB. XIX.

To cut off from the Triangle A B C, *a Triangle at* B *equal to a given one* B D E. Fig. 41.

Make at the Point D the Angle B D F equal to the given Angle A B C, draw E F parallel to B D and F B to E D, take B G equal to D F and B H equal to B D: draw G H, which will cut off the Triangle B G H equal to the given one B D E.

Quadrilaterals may be easily divided by one who understands the Division of Triangles, tho' the Division of Triangles depends, in several Cases, upon the Division of Quadrilateral Figures, and principally Trapezia; as, when a Triangle is to be divided into more than two equal Parts.

PROB. XX.

To make a scalene Triangle A B C, *equal to a right angled Triangle.* Fig. 42.

Draw B D parallel to the Base A C, from A erect the Perpendicular A D of the same Height with the Parallel, and draw C D. The Triangle A D C is a right angled Triangle, and equal to the Triangle A B C.

PROB. XXI.

To make a Scalene Triangle A B C *equal to an Isoceles.* Fig. 43.

Draw B D parallel to A C, make A E equal to E C; from E raise the Perpendicular E F and draw A F and CF. The Isoceles Triangle E F C is equal to the Scalene A B C.

PROB.

PROB. XXII.

Fig. 44. *To make a Scalene Triangle* A B C *equal to an equilateral one.*

Upon A C make an equilateral Triangle A D C; upon A D deſcribe a Semicircle A F D, draw B E parallel to A C. From E raiſe the Perpendicular EF cutting the Semicircle in F, draw A F upon which make the equilateral Triangle A G F, which is equal to the ſcalene one.

PROB. XXIII.

Fig. 45. *To find the Center of a given Circle.*

Draw a Line BC in the Circle at Pleaſure, which biſect in Q; thro' I, draw the Perpendicular L F, which biſect in A. A will be the Center.

PROB. XXIV.

Fig. 46. *To deſcribe a Circle, whoſe Area ſhall be double to that of a given one.*

Divide the Circle A B C D into four equal Parts, take the Diſtance AB and put it from E to F, and with the Diſtance EF deſcribe a Circle, whoſe Area will be double to the Area of the given Circle.

Hence, the Area of a Circle may be made three, or four times, &c. bigger than the given one.

PROB. XXV.

Fig. 47. *To deſcribe a Circle that ſhall paſs thro' three Points, not lying in a ſtrait Line, as the Points* A, B *and* C.

Join the Points B A and B C with right Lines; divide the Line A B into two equal Parts, and draw the Line XY. Alſo divide the Line B C into two equal Parts, and draw the Line Z Y, and at Y the Point of Interſection will be the Center of the Circle, which will paſs thro' the Points A, B and C.

Hence, any Triangle may be inſcribed in a Circle, by biſecting its Legs AB and BC.

Hence, one Arch of a Circle may be perfected, by drawing two Lines A B and B C *within the given Arch, and biſecting them.*

Hence, the Center of a Circle may be found, by drawing within the Circles two Lines A B *and* B C, *and biſecting them.*

PROB.

PROB. XXVI.

To make an Oval. Fig. 48.

Draw a Line, and thereupon deſcribe a Circle, BAC; from C deſcribe another Circle CBD thro' B: From the Point of Interſection E, draw the Lines EH and EG thro' the Centers B and C; from E withthe Diſtance EG deſcribe an Arch from G to H, and from the Point F with the ſame Diſtance deſcribe an Arch from I to K, and ſo the Oval is finiſhed.

PROB. XXVII.

To make a right lined Triangle equal to a given Circle. Fig. 49.

Divide the Diameter AB of the given Circle into ſeven equal Parts; erect a Perpendicular on B thrice as long as the Diameter AB, and a ſeventh Part over as BC: Draw a right Line from C to D the Center of the given Circle, and the Triangle DCB, is that which was to be done.

Hence the Circumference of a Circle may be known by having the Diameter given.

PROB. XXVIII.

From a Point C *given in the Circumference to draw a Tangent* XY. Fig. 46.

From the Center E thro' the given Point C draw the Line EZ, ſo that CZ may be equal to EC, biſect EZ in C with the Line RS, which will be the Tangent required.

PROB. XXIX.

To draw a Spiral Line. Fig. 50.

Upon a ſtreight Line XY deſcribe an Arch ABC; from the Point C with the Diſtance BC deſcribe another Arch touching the ſame Line in D; from the firſt Center A deſcribe another Arch from D that ſhall cut the Line in E: Return to the Point B, and from B as the Center with the Diſtance, deſcribe another Arch to F. Take A as the Center, and deſcribe an Arch from F to G; and ſo on to Infinity.

PROB. XXX.

To make a Square ABCD. Fig. 11.

Upon B raiſe a Perpendicular B C of the ſame Height, with the ſame Diſtance make two Arches from A and C, interſecting each other at D and join D A and A C by right Lines, which will compleat the Square A B C D.

PROB. XXXI.

Fig. 51. *To make a Square* D F, *equal to a given Circle.*

Divide the Diameter A B into ſeven equal parts, double the Diameter, and add a ſeventh part of itſelf to it, as, A C; divide the firſt Diameter A B equally in D, and divide the Line D C equally in E, with the Diſtance D E deſcribe an Arch D F C. Upon E raiſe the Perpendicular E F touching the Circle; this Line E F will be a Side of the Square required.

PROB. XXXII.

Fig. 53. *To inſcribe in a Circle a Triangle* D, *having equal Angles with a given one* X.

Let the Line E F touch the Circle in D, make EDG equal to Angle C, and F D H equal to B, and join G H. The Thing is done.

A right lined Figure is ſaid to be inſcribed in a Circle, when all the angular Points of the Figure touch the Circle's Periphery.

PROB. XXXIII.

Fig. 54. *To inſcribe a Circle in a Triangle.*

Biſect the two Angles with the Lines C A and E A meeting together in A, which will be the Center of the Circle to be inſcribed.

PROB. XXXIV.

Fig. 55. *To inſcribe a Square in, and circumſcribe one about a Circle.*

Draw the Diameters B D and C E cutting one another perpendicularly. The Lines joining the Terms of theſe, inſcribe a Square in a Circle. Then draw the four Tangents touching the Circle in B, C, D and E meeting together in I, F, G and H. The Figure I F G H is a Square circumſcribed about a Circle.

PRO

PROB. XXXV.

To make a Square two, three or four times greater than it is. Fig. 56.

Extend the Side AB, take the Distance BD, and set upon the extended Side AB, from B to E, and it will be the Side of a Square double to the given one ABCD, and so on.

PROB. XXXVI.

Three or more Squares being given, whose Sides are AB, BC *and* CE, *to make one Square equal to them all.* Fig. 57.

Make a right Angle Triangle FBZ, having indefinite Sides, and upon the Sides of it tranfer the Lines AB and BC, then join AC. The Square of AC will be equal to the Squares AB and CB together. Then transfer AC from B to X, and CE, the third given Side, from B to E, and join EX, the Square of EX shall be equal to the three given Squares AB, BC and CE.

PROB. XXXVII.

To make a Square equal to an oblique angled Parallelogram. Fig. 58.

From the Points A and B let fall the Perpendiculars on CD, to AB add BX equal to BF, and divide AX equally in G, and with the Distance AG describe a Semicircle ASX. From B erect a Perpendicular touching the Semicircle in S, which will be the Side of the Square required.

PROB. XXXVIII.

To make a Parallelogram with an Angle in it equal to a given one O, *and equal to a given Triangle* ABC. Fig. 59.

Bisect the Base AB in F, thro' C draw CX parallel to AB, make the Angle BAL equal to the given one O, and draw FI parallel to AL. ALIF will be the Parallelogram required.

PROB. XXXIX.

To inscribe a Parallelogram in a Circle, whose Length is to the Breadth, as five to two. Fig. 60.

 Divide

Divide the Diameter X Y into five equal Parts; set off two of them from Y to Z, from X thro' Z draw X Z S the Length of the Parallelogram, and from S draw the Breadth of the Parallelogram required.

Hence a Parallelogram may be inscribed in a Semicircle, whose Length is to its Breadth, as five to two.

PROB. XL.

Fig. 61. *To reduce a Square to an equilateral Triangle.*

Upon d z the Side of the Square, take z x at pleasure, and with that Distance describe the Arch x o n, upon x z make the equilateral Triangle x o z. Set off one half of its perpendicular Altitude from z to y, bisect x y in t, on which describe the Semicircle x m y; m z, is a mean Proportional betwixt x z, the Base of the equilateral Triangle x o z, and half its Perpendicular z y. The Square of m z is equal to the equilateral Triangle x o z draw o m, parallel to which draw B h till it meet z o produced to h, and on z h make an equilateral Triangle p h z, which will be equal to the given Square. For the Triangles z o m and z B h are similar, by Construction; then, as z m : z o : : z B : z h : that is as the mean Proportional z m is to the Side of its correspondent equilateral Triangle x o z, so is the mean Proportional z B, the Side of the given Square, to the Side of the equilateral Triangle p h z, which is epual to the given Square.

Hence, all regular and irregular Polygons, Trapeziums, Parallelograms and Triangles may be reduced to equilateral Triangles, or to any right lined Triangle, if the Angles are given severally; first by reducing them to a Square, and then finding a Triangle equal to that Square.

PROB. XLI.

Fig. 62. *To make a Square equal to a Triangle whose Angles are all known.*

Upon the Side C D, make a Triangle C o D, including the Angles of a given Triangle, set off half its Perpendicular r o from C to p, on D p, describe a Semi-

a Semicircle D o p, then the Square of C S, will be equal to the Area of the Triangle D O C. Draw o s, parallel to which from B draw B n till it meet O C produced to n: From n parallel to O D draw n t meeting D C produced to t. Set off half the Perpendicular n m of the Triangle t n c, from C to L: From m the Center describe a Semicircle. The Triangle t n c will be equal to the Square, and the Triangles are similar.

Hence, a Triangle whose Area and Angles are given, may be geometrically constructed, if a Side of a Square be found equal to the given Triangle.

PROB. XLII.

Upon a given Line R S, *to describe a Polygon like, and in like manner posited to a given one,* B Q. Fig. 63.

Resolve the given Polygon B Q into Triangles, upon the given Line R S make the Angles R and O equal to the Angles A and B, (by Problem 7.) the Sides will then meet together in X: Upon X S make the Angles V and I equal to the Angles T and C: The Sides will then meet together in Z. The Thing is done.

Hence, is derived the Method of making Maps, or Charts, whether Geographical or Chorographical, or those which Surveyors of Land make; and of framing Ichnographical Delineations of Fields, Buildings, and Countries: For they are only but the Reduction of great Figures unto like Figures of a smaller Compass, and is performed by this Problem.

PROB. XLIII.

Upon a given Line to describe a regular Pentagon. Fig. 64.

Make the given Line A B Radius, and upon each End describe a Circle, and thro' the Points X an G of Intersection draw the Line GEX. From G, with the same Radius, describe an Arch H E D, lay a Ruler to the Points D and E, and mark where it crosses the other Circle, as F, also to to the Points H and E, and mark where it crosses the other Circle, as C. From the Points F and C, with the same Radius, describe two Arches cutting one another in K, join the Points AF, KC, F K, and C B with right Lines, and they will form the Pentagon required.

PROB. XLIV.

Fig. 65. *To describe a regular Pentagon in a Circle.*

Draw the Diameters E D and B F cutting one another perpendicularly in A. Bisect the Radius AD in C. From the Center C thro' B describe an Arch, including the Diameters E D in G. The Line D G is the Side of a Pentagon, and H G of a Decagon.

Hence a Circle may be divided into five, ten, twenty, &c. equal Parts.

PROB. XLV.

Fig. 66. *To reduce a Triangle* ABC *to a Pentagon* D E F G C.

Reduce the Triangle ABC to a Square EXDC, with any Distance on BC, as C y, describe the Circle k z C, in which make a Triangle k y C, and set off one half of its Perpendiecular L y, from C to T on k T, from S describe the Semicircle k y T. So the Square of n C, will be equal to the Triangle k y C, draw y n; parallel to which draw x m till it meet BC produced, and I m, parallel to k y, then the Triangle I m C, will be the Triangle of a Pentagon, and equal to the Triangle ABC; from B let fall the Perpendicular B y, and set off the one tenth of B y from C to u; then from h with the Distance I h, describe the Semicircle I n u. Draw F i parallel to y n, and F y parallel to y k; with the Distance F C, describe a Circle C G X, and with the Distance q C describe the Pentagon q C O P G, which was to be done.

Hence, any right lined Figure can be reduced to any regular Polygon, and any regular Polygon to any other Polygon, or to any right lined Triangle, if the Angles are given.

PROB. XLVI.

Fig. 67. *To reduce a Pentagon* ABCDE, *to an irregular Polygon*

Reduce the Pentagon ABCDE, to the Square GE. Draw a Polygon a b c o d E similar to that required, and reduce it to a Square, (by *Euclid.* l. ii. Prop.

Prop. 14) of which make N E one Side, draw N O and H e parallel to it. Draw e P, c M, M h, h t parallel to od, oc, c b, ba. So the Polygon t h m e P E t, will be equal to the Square G E, which is equal to the Pentagon, for it is as N E : H E : : o E : e E, (by *Euclid.* l. vi. Prop 4.) (by Prop. 19.) as the Square of N E : H E : : o E : e E.

Hence, one irregular Polygon can be reduced to another irregular one.

Hence also, all right lined Figures can be divided into extream and mean Proportion; for if any Side of the Figure be divided into extream and mean Proportion, and on each Segment be drawn a Polygon similar to that given, these Polygons will be directly as their Sides.

PROB. XLVII.

Upon a given Line to raise a regular Hexagon. Fig. 68.

Make on B C an equilateral Triangle C A B; from the Center A thro' B and C, describe a Circle. This will contain a Hexagon upon the given Line C B.

PROB. XLVIII.

To inscribe a regular Hexagon in a given Circle. Fig. 69.

Draw the Diameter F A B; from the Center B, thro' A describe a Circle cutting the given one in C and D; also from F thro' A, a Circle cutting the given one in E and G. The six Points B, C, E, F, G and D, joined by right Lines, will give the Hexagon required.

Hence, a Circle can be divided into twelve, twenty-four, forty-eight, &c. equal Parts.

The Side of an Hexagon inscribed in a Circle, or a Chord of sixty Degrees, is equal to the Radius, and consequently the Sign of thirty Degrees is equal to half the Radius.

An Angle of a regular Hexagon is four thirds of a right Angle, as consisting of two Angles of an equilateral Triangle, each of which makes two thirds of a right Angle.

An equilateral Triangle is easily inscribed in a Circle, by drawing the Diameter F B, *and describing from the*

the Center B *thro'* A *the Arch* CAD. *The Points* CFD *joined with right Lines, will give the Triangle sought.*

PROB. XLIX.

To inscribe a regular Quindecagon in a Circle.

Fig. 70. Inscribe in the Circle AC the Side of a Pentagon, and AD the Side of an equilateral Triangle, bisect the Arch CD in E, CE is the Side of the Quindecagon sought.

For if the Circumference be supposed to be fifteen, the Arch AC will be three, and the Arch AD five, therefore the Arch CD two, and consequently CE one.

Hence innumerable regular Figures may be inscribed in Circle; for if AC, *and* AD *the Sides of the two regular Figures be inscribed in a Circle, the Difference of the Arches* CD, *will contain so many Sides of a new regular Figure as the Units, are whereby the Denominators of the former differ one from another. But the Denominators of the new Figure is had, if the Denominators of the former be multiplied one by another. As, if* AD *be the Side of a Square, and* AC *of a Decagon, the Difference of the Denominators is six. Therefore the Arch* CD *contains six Sides of a new Figure. But the new Figure is of forty Sides. For the Denominators* 4 *and* 10, *multiplied the one by the other make* 40.

PROB. L.

Fig. 70. *To inscribe regular Figures* 7, 9, 11, 13, 17 *Sides &c. in a Circle.*

Divide the whole Circumference, *viz.* 360, by the Denominator of the Polygon to be inscribed, *e. g.* a ninesided Figure. Make at the Center the Angle AKG of so many Degrees as there are Units in the Quotient, which will be 40; AG will be the Side of the nine angled Figure required to be inscribed in the Circle.

PROB. LI.

Fig. 70. *Upon a given right Line any regular Figure whatever may be described by the following Table.*

A right Angle is to the Angle of the Figure.

In	as	to Difference
A Pentagon	5	6—1
An Hexagon	3	4—1
An Heptagon	7	10—3
An Octagon	2	3—1
A Nonagon	9	14—5
A Decagon	5	8—3
An Undecagon	11	18—7
A Duodecagon	3	5—2

Describe a regular Heptagon on the given Line XB. From the Center X, with the Distance XB, describe a Circle, from which cut the Quadrant BO. See in the Table the Proportion of a right Angle to the Angle of an Heptagon, which will be as 7 to 10, and the Difference 3. Therefore divide the Quadrant into 7 equal Arches, so many of which add to it from O to N as the Difference has Units. Thro' the three Points B, X and N describe a Circle; this contains an Heptagon on the given right Line XB.

PROB. LII.

To make a Cylinder equal to a Parallelepiped of the same Height. Fig. 16, & 15.

Upon the Base of the Cylinder CBAD make a Square ABCD, upon which raise four Perpendiculars of the Height of the Cylinder AB, as E, F, G and H, and join EF, EH, FG and GH by right Lines, which will make the Parallelepiped required.

PROB. LIII.

To make a Parallelepiped equal to a Cylinder. Fig. 15, & 16.

Make a Circle equal to the Base of the Parallelepiped, and raise this Circle to the same Height with the Parallelepiped, and the Work is done.

PROB. LIV.

To make a Cone equal to a Pyramid of the same Height. Fig. 18, & 17.

Make a Triangle, Square, Pentagon or any other Polygon equal to the Base of the Cone CBD, and make it the Base of the Pyramid, as, ABCD; raise, from

from the Middle of the Baſe F, a Perpendicular F E of the ſame Height with the Cone B A, then draw from A B C D Lines to the Point E, and the Pyramid is made.

P R O B. LV.

Fig. 17, & 18. *To make a Pyramid equal to a Cone.*

Make a Circle equal to the Baſe of the Pyramid, raiſe upon its Center a Perpendicular B A of the ſame Height with the Cylinder, and join the Extremities of the Diameter C D till they meet at the Top of the Perpendicular A, and the Cone will be finiſhed.

P R O B. LVI.

Fig. 16, 17, 18. *To make a Priſm or Cylinder equal to a Pyramid or Cone of the ſame Height.*

Make the Baſe of the Cylinder three times as much as it is; upon its Center raiſe a Perpendicular of the ſame Height with the Cylinder, and join the Extremities with the Top of the Perpendicular, and the Pyramid will be made.

In like Manner a Cylinder or Priſm may be changed into a Cone.

P R O B. LVII.

To make a Pyramid or Cone equal to a Priſm or Cylinder.

Reduce the Baſe of the Pyramid or Cone to one third of the Baſe of the Priſm or Cylinder, and on that Baſe erect a Pyramid or Cone of the ſame Height with the Priſm or Cylinder given.

P R O B. LVIII.

To make a Cube equal to a Parallelepiped.

If the Baſe of the Parallelepiped be a Square, find a mean Proportional between its Height and one Side of the Baſe; this mean Proportional, will be the Meaſure of the Cube required. If the Baſe be a Parallelogram, make it equal to a Square; then go on as at firſt till the Work is finiſhed.

P R O B

PROB. LIX.

To make a Cube equal to a given Cylinder.

Make a Parallelepiped equal to the given Cylinder. then make a Cube equal to a Parallelepiped.

PROB. LX.

To make a Cube equal to a given one.

Make a Parallelepiped equal to a given Cone, then make a Cube equal to that Parallelepipid.

PROB. LXI.

To make a Cone equal to a Globe. *Fig.* 20.

Make a Circle four times bigger than one of the great Circles of the given Globe, raiſe a Cone upon this Baſe of the ſame Height with the Semidiameter A B of the Globe. The Cone C D E will be equal to the Globe A B D.

PROB. LXII.

To make a Cube equal to a given Globe.

Make a Cone equal to the given Globe, then make a Parallelepiped equal to this Cone, and a Cube equal to this Parallelepiped.

THE

ELEMENTS

OF

Plain TRIGONOMETRY.

SECT. I.

Of the Definitions and Affections of Triangles, and of Constructing the CANON *of Natural and Artificial* SINES, TANGENTS *and* SECANTS.

DEFINITIONS.

PLAIN TRIGONOMETRY, is the Art whereby, from any three given Parts of a plain Triangle, we find all the rest.

Fig. 1. *Thus, e. g. from two Sides* AB *and* AC, *and an Angle* B, *we find by* TRIGONOMETRY, *the other Angles* B *and* C *with the third Side* BC.

Fig. 2. A *Chord* of an Arch or Angle, is a right Line AB dividing the whole Circle into two parts, and subtends both Segments.

Hence, the greatest Chord that can be drawn in a Circle, is the Diameter.

Hence also, all the Chords of Arches greater than a Semicircle, are less than the Diameter.

A C

TRIGONO

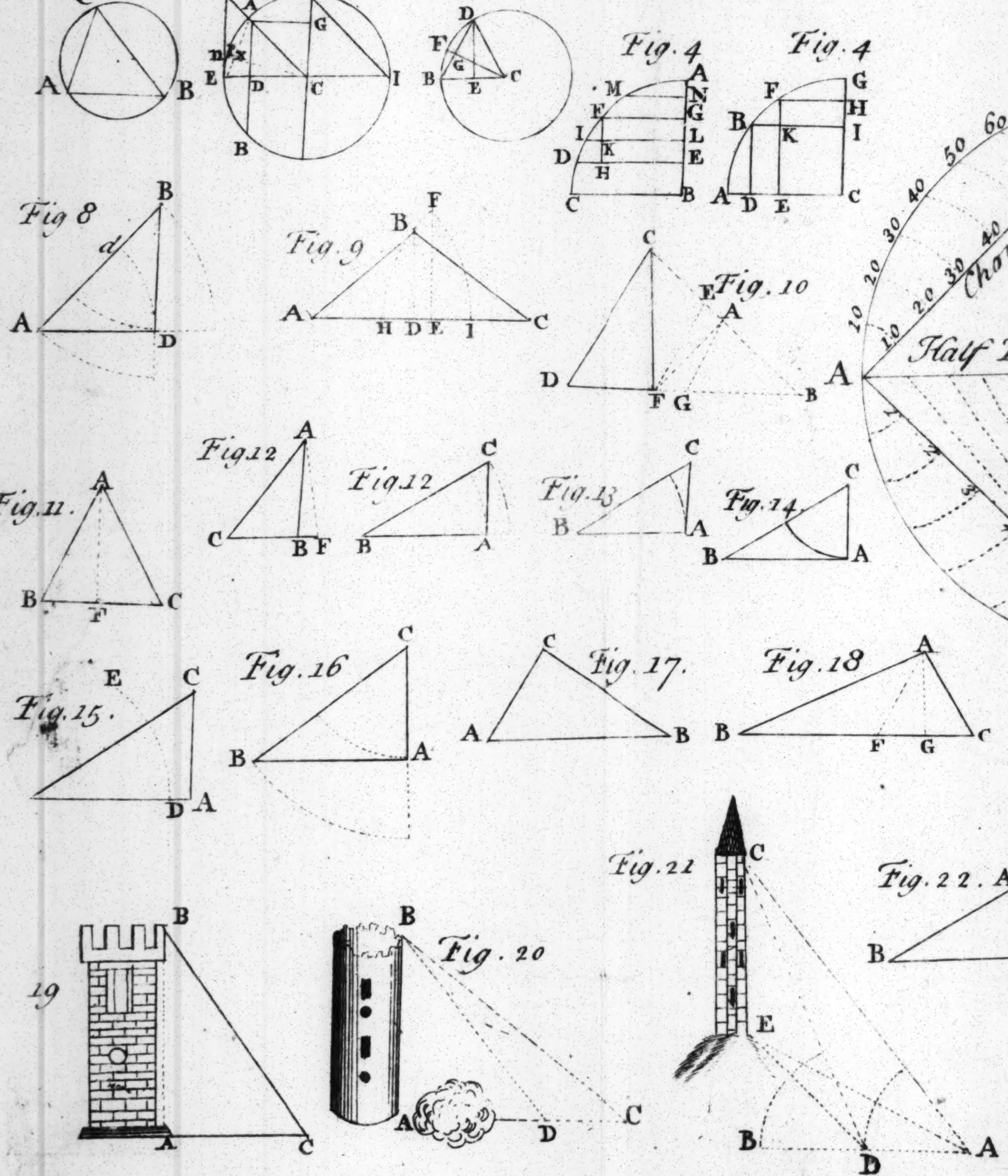


Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 4
Fig 8
Fig. 9
Fig. 10
Fig. 11.
Fig. 12
Fig. 12
Fig. 13
Fig. 14
Fig. 15.
Fig. 16
Fig. 17.
Fig. 18
19
Fig. 20
Fig. 21
Fig. 22.
Half

ONOMETRY

Fig. 6

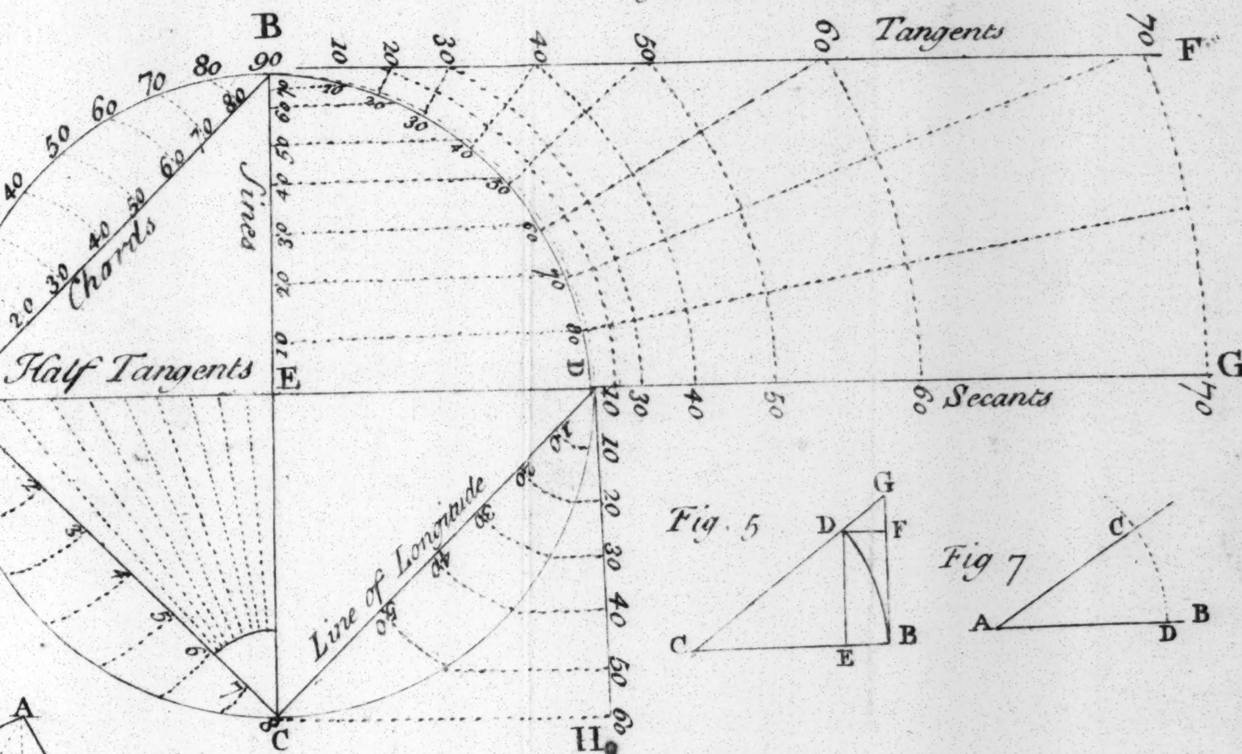
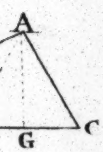


Fig. 5

Fig 7



ig. 22.

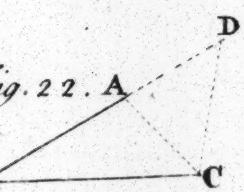


Fig. 24.

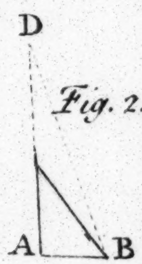


Fig. 25.

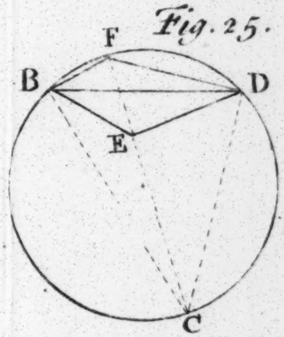


Fig. 26.

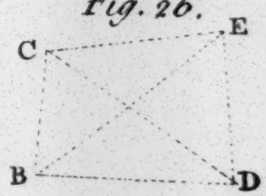


Fig. 23.

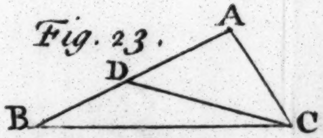
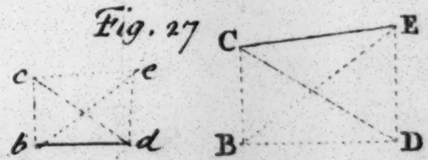


Fig. 27



A *ri*
one ha
or A I
Henc
E C, *c*
Radius
A *w*
the Qu
A *ve*
D I, in
the Ar
Henc
Since
A C E,
A C I;
Angle ;
verſed S
whole S
There
ſame Si
which
A *Ta*
ouchin
ight A
C, ca
F E
ngle A
Theref
nt and
The
the T
the A
her A
The C
ants of
. *Thi*
a Qua
Degr
ees, is
the C
The F
C, ma

A *right Sine* A D, of the Arch A E or A I, is one half of the Chord A B of the double Arch A E B or A I B.

Hence, the Sine A D *is perpendicular to the Radius* E C, *consequently, all Sines standing upon the same Radius are parallel between themselves.*

A *whole Sine*, is the Radius H C, or the Sine of the Quadrant H E.

A *versed Sine*, is that Part of the Radius E D, or D I, intercepted betwixt the right Sine A D, and the Arch A E or A I.

Hence, the greatest versed Sine is the Diameter E I.

Since that the Arch A E *is the Measure of the Angle* A C E, *and* A I *is the Measure of its contiguous Angle* A C I; *but the Quadrant* H E *is the Measure of a right Angle*; A D *will also be the right Sine, and* E D *the versed Sine of the Angles* A C E *and* A C I; *but the whole Sine is the Sine of a right Angle.*

Therefore two Angles, which are adjacent, have the same Sine. Likewise, obtuse Angles have the same Sines, which their Complements have to two right ones.

A *Tangent* of an Arch A E is a right Line, E F touching the Circumference of the Circle, and is at right Angles to the Diameter E I, and limited by F C, called the *Secant* of the same Arch.

F E *is also the Tangent, and* F C *the Secant of the Angle* A C E, *and also of the Angle* A C I.

Therefore two adjacent Angles have the same Tangent and Secant.

The *Cosine*, is the Sine A G, the *Cotangent* F H the Tangent, and the *Cosecant* F C is the Secant of the Arch A H, which is the Complement of the other Arch A E to a Quadrant.

The Complement of an Arch, or Angle, is what it wants of a Quadrant, a Semicircle, or of a whole Circle. Thus 20 *Degrees is the Complement of* 70 *Degrees to a Quadrant; because* 20 *Degrees is the Remainder of* 70 *Degrees subtracted from* 90 *Degrees: Also,* 50 *Degrees, is the Complement of* 130 *to* 180 *Degrees, and* 70 *the Complement of* 290 *to* 360 *Degrees.*

The Radius C A, with the Sine A D and Cosine D C, make a Triangle C A D similar to the Trian-

gle CFE made by the Radius CE, Tangent EF and Secant CF. Likewise, the Radius, Cotangent and Cosecant make another Triangle similar to the two former.

Hence, as the Cosine is to the Sine, so is the Radius to the Tangent. That is, as CD : AD :: CA : EF.

As the Radius is to the Sine, so is the Secant to the Tangent. That is, as CA : AD :: CF : FE.

As the Sine is to the Radius, so is the Radius to the Cosecant. That is, as DA : CA :: HC : CF.

As the Tangent is to the Radius, so is the Radius to the Cotangent. That is, as FE : EC :: CH : FH.

Therefore the Rectangle, between the Tangent and Cotangent of any Arch, is equal to the Square of the Radius.

When a Triangle is given to be resolv'd, first, we are to consider, that there is in the Table of *Logarithms*, Sines, Tangents, and Secants, a Triangle exactly similar and equal to that which is to be solved, and whose Sides are to one another in the same Proportion of those of the Triangles proposed.

Next, we must understand whatever *Ratio* one Side of the given Triangle has to the other Side about the same Angle, considered as Lengths estimated or numbered by any known Measure: As suppose, Inches, Yards, Miles, *&c.* the very same has the two Sides about the same Angle, in the Triangles in the Tables, or in the tabular Parts; which two Things, well understood, will lead us into the whole Mystery of Trigonometrical Calculations.

In estimating the Quantity of Sines, &c. we assume Radius for Unity; and determine the Quantity of Sines, Tangents and Secants in Fractions thereof. From Ptolomy's Almagest, *we learn, that the Antients divided the Radius into 60 Parts, which they called Degrees; and thence determined the Chords in Minutes, Seconds and Thirds; that is, in sexagesimal Fractions of the Radius, which they likewise used in resolving Triangles. The Sines, or half Chords, were first used by the* Saracens. Regiomontanus, *first, with the Ancients, divided*

divided the Radius into sixty Degrees, and determined the Sines of several Degrees in decimal Fractions thereof. But he afterwards found it would be more commodious to assume Radius for one; and thus introduced the present Method into Trigonometry. *In common Tables of Sines and Tangents, the Radius is supposed to be divided into* 10,000,000 *Parts, beyond which we never go in determining the Quantity of Sines and Tangents. Hence, as the Sine of a Hexagon subtends the sixth Part of a Circle, and is equal to the Radius, the Sine of* 30 *Degrees is* 5,000,000.

Trigonometrical Problems.

PROB. I.

The Sine AD *being given, to find the Cosine, or Sine Complement,* AG. *Fig.* 2.

BECAUSE that EC, the Sine of the same Arch EH, is perpendicular to HC and AG; the Sine of the Arch AH is perpendicular to the same HC; AG will be parallel to DC, and the Angle AGC a right Angle, and so AGC will be a right angled Triangle. Wherefore, seeing AD and HC are Perpendicular to EC; GC will be equal to AD. If therefore from the Square of the Radius AC be subtracted the Square of the Sine AD, or GC, the Remainder will be the Square of the Cosine AG. Whence if the square Root be extracted, it will give the Cosine AG, *e. g.* Let AC be 10,000,000, AD 5,000,000, AG will be 8,660,254, the Sine of 60 Degrees.

PROB. II.

The Sine AD *of the Arch* AE *being given, to find the Sine of half that Arch.* *Fig.* 2.

Find the Chord of the Arch AE; for half of this is its Sine. Thus, *e. g.* DG and AD, as in the preceeding Problem; we shall find the Sine of half the Arch AE, or the Sine of 15 Degrees =2,588,190.

PROB. III.

Fig. 3. *The Sine* DG *of the Arch* DF *being given, to find the Sine* DE *of the double Arch* DB.

Since the Angles at E and G are right Angles, and the Angle B is common to both Triangles, BCG and DEB; BC : CG :: BD : DE. Wherefore CG being found by the second Problem, and BD deing double of DG; DE is found by the Rule of Proportion.

Hence, CB : 2CG :: BD : 2DE, *that is, the Radius is double to the Cosine of one half of the Arch* DB, *as the Subtense of the Arch* DB *is to the Subtense of double the Arch. Also, as* CB : 2CG :: | 2BG : 2DE :: | BG : DE :: $\frac{1}{2}$CB : CG. *Wherefore, the Sine of any Arch, and the Sine of its Double being given, the Cosine of the Arch itself is given.*

PROB. IV.

Fig. 4. ***The Sines* FG *and* DE *of the Arches* FA *and* DA, *whose Difference* DF *is not greater than* 45 *Minutes, being given, to find any intermediate Sine, as* IL.**

To the Difference FD of the Arches, whose Sines are given; the Difference of the Arch IF, whose Sine is required, and the Difference of the given Sines DH, find a fourth Proportional: This added to the less given Sine FG, the Aggregate will be the Sine required.

PROB. V.

The Sines BD *and* EF *of the two Arches* AB *and* AF, *being given, to find the Sine* BF *of the Arch of half the Difference.*

Subtract the lesser Sine BD from the greater EF, and the Remainder will be FK. From the given Sines BD and EF find the Cosines BI and FH, by Problem I, subtract the lesser Cosine FH from the greater BI, the Difference will be BK. Extract the square Root from the Sum of the Difference of the Squares, the Remainder will be BF, the half of which is the Sine sought.

PROB.

PROB. VI.

To find the Sine of 45 Degrees. Fig. 2.

Let HI be a Quadrant of the Circle, then will HCI be a right Angle; consequently the Triangle, Rectangular: Therefore $HI^2 = HC^2 + CI^2 = HC^2$; wherefore, since HC the whole Sine, is 10,000,000; if from $2HC^2$ squared 200,000,000,000,000, be extracted, the square Root 14,142,136, the Chord HI will be the Remainder, whose half 7,071,068, the Sine of 45 Degrees required.

THEOREM VII.

In small Arches, the Sines and Tangents of the same Arches are nearly to one another, in a Ratio of Equality. Fig. 5.

The Triangles CED and CBG being equiangular, CE : CB : : ED : BG; but as the Point E approaches B, EB will vanish in respect of the Arch BD. Whence CE will become nearly equal to CB. And so ED will also be nearly equal to BG. If EB be less than the $\frac{1}{10\,000\,000}$ Part of the Radius, then the Difference between the Sine and the Tangent will also be less than the $\frac{1}{10\,000\,000}$ Part of the Tangent.

Since any Arch is less than the Tangent, and greater than its Sine, and the Sine and Tangent of a very small Arch are nearly equal, it follows, that the Arch will be nearly equal to its Sine; and so in very small Arches it will be, as Arch is to Arch, so is Sine to Sine.

PROB. VIII.

The Sine of one Minute or 60″ FG being given, to find the Sine of one or more Seconds MN. Fig. 4.

Since the Arches AM and AF are very small, AMF may be taken for a right Line, without any sensible Error in the Decimal Fractions of the Radius, wherein the Sine is expressed; that is, the Arches AM and AF, may be taken proportional to their Chords. Wherefore, since MN is parallel to FG; AF : FG : : AM : MN. Therefore AF, FG and AM being given, MN is easily had.

PROB. IX.

To find the Sine of the Arch of one Minute.

The Subtenſe of 60 Degrees is equal to the Radius, ſo the half of the Radius will be the Sine of the Arch of 30 Degrees. Wherefore, the Sine of the Arch of 30 Degrees being given, the Sine of the Arch of 15 Degrees may be found, (by Prob. II.) and ſo on till twelve Biſections being made, we come to an Arch of 52^{2}, 44^{3}, 03^{4}, 45^{5}, whoſe Coſine is nearly equal to the Radius, in which Caſe the Arches are proportional to their Sines: And ſo, as the Arch 52^{2}, 44^{3}, 03^{4}, 45^{5}, is to an Arch of one Minute, ſo ſhall the Sine before found be to the Sine of one Minute; and when the Sine of one Minute is found, then the Sine and Coſine of two Minutes will be had.

PROB. X.

Fig. 2. *The Sine* A D *of the Arch* A E *being given, to find the Tangent* E F, *and the Secant* F C *of the ſame Arch.*

Becauſe the Sine AD and Tangent EF are perpendicular to the Radius E C, A D will be parallel to E F: Wherefore, as the Coſine D C is to the Sine AD, ſo is the whole Sine to the Tangent E F: Allſo, as the Coſine DC is to the whole Sine A C, ſo is the whole Sine E C to the Secant C F.

PROB. XI.

To conſtruct a Canon of Sines.

The Sines of 30°, 15°, 45° and 36°, (which we have already ſhewn how to find) being had, we can thence conſtruct a Canon of all the Sines to every Minute, or even a Second For from the Sine of 36°, we find thoſe of 18°, 9°, 4° 30 and 2° 15 (by Prob. II.) the Sines of 54°, 72°, 81°, 85° 30 and 87° 45′, &c. (by Prob. I.) Again, for the Sine of 45° find the Sine 22° 30′, 11° 15′, &c. From the Sines of 30° and the Sines of 54° find the Sine of 12°. From the Sine of 12°, find the Sines of 6°, 3°, 1° 30, 45 78°, &c. From the Sine of 15° find the Sine of 7° 30′ 30° 45′, &c. till you have 120 Sines ſucceeding each other orderly at

an

an Interval of 45 Minutes. Between these find the intermediate Sines (by Prob. V.) Thus will the Canon be compleat.

PROB. XII.

To find the Logarithm of any given Number.

The first Page of the annexed *Tables of Logarithms* contains all the natural Numbers in their proper Order, from 1 to 100. And against every one of these Numbers is placed its *Logarithm*, with its *Index* before it.—Thus against the Number 28, its Log. is 1.447158; and against the Number 89, its Log. is 1.949390: And so on for the rest. In the first Column of all the following Pages, under *Num.* the natural Numbers proceed in their due Order, from 100 to 1000. And in the next Column, under 0, against every one of these Numbers, is the Decimal Part of its Logarithm, without any Index; to which its proper Index must be prefixed, according as the natural Number used requires, *e. g.* against the Number 856, under 0, is 932474; to which if 2 the Index of 856 be prefixed, it will be 2.932474, the compleat Logarithm of 856.

The other five Columns of each Page contain the Logarithms of all Numbers, from 1000 to 10000. Those in the Left-hand Pages are distinguished on the Top of the Columns with the Figures 0.1.2.3.4, and those in the Right hand Pages with 5.6.7.8.9. So that to find the Logarithm of any Number between 1,000 and 10,000, as suppose of 5,468 seek for the three first Figures, *viz.* 546, in the first Column under *Num.* and for the last Figure, *viz.* 8 at the Top. Then in the Column under the last Figure 8, and over-against the three first Figures 546, there is 737829; to which if 3, the Index of 5,468, be prefixed, the compleat Logarithm thereof will be 3.737829: and so for any other Logarithm of any proposed Number, not exceeding 10,000. But if the proposed Number be above 10,000, *which is the Limits of the annexed Table*, then the Logarithm of that Number must be found, by the Help of the common *Difference* of the Logarithms, which is the last Column of every Page under *Diff.* Thus: *Find*

Find the Logarithms of the first four Figures of the given Number, without its Index, as above; and multiply the common Difference which stands against the Logarithm, under Diff. *with the other Figures of the given Number, casting off so many Figures of that Product as there are in the Multiplicator; then add the remaining Figures of that Product to the Logarithm of the first four Figures, and to their Sum prefix the proper Index; and you will have the compleat Logarithm required.*

Suppose it were required to find the Logarithm of 698,476. First, the Logarithm of 6,984 is found in the Table, as above, to be 844,104, and against it, under *Diff.* is 62. This 62 being multiplied with 76, the other two Figures of the given Number, produces 4,712. Cut off 12, *viz.* the two last Figures, and then add 47 to the Logarithm last found, and the Sum will be 844,151; to which prefixing 5, the proper Index of the given Number 698,476, it will be 5.844151, the Logarithm required.

PROB. XIII.

To find the Number to any given Logarithm.

Omit the Index of the given Logarithm, and then seek it in the Table of Logarithms, and if exactly found there, then the Number in the first Column under *Num.* with that on the Top over Logarithm, will be the Number required. But if the given Logarithm, without its Index, cannot be exactly found in the Table, then the proper Number agreeing to that Logarithm may be found by the Help of the common Difference of Logarithms: Thus:

From the given Logarithm subtract the next less, and to the Remainder annex Cyphers; then divide it by the common Difference found against the next Logarithm, under Diff. *and the Quotient, will be a Number that must be annexed to the Number already found against the next less Logarithm, according as the Index of the given Logarithm denotes.*

Suppose, 5.660279 were a given Logarithm, and it were required to find the natural Number answering it.

The

The Number ſought muſt conſiſt of 6 Places of Figures in whole Numbers, as appears by its Index 5; which being omitted, ſeek in the Table of Logarithms for 660279; but not finding it exactly there, take the next leſs to it, *viz.* 660201 ſtanding under 3, and againſt 547: Therefore, the firſt four Figures of the Number ſought muſt be 4,573, and the common Difference found againſt 660201, under *Diff.* is 95.

Then for the Logarithm	660279
Subtract the next leſs, *viz.*	660201
Remains	78

To which annex two Cyphers, becauſe there is yet wanting two Places of Figures, and it will be 7,800, which being divided by the common Difference 95, the Quotient will be 82, which muſt be annexed to 4,573, and the Sum will be 457,382, the Number anſwering to the given Logarithm 5.660279. Thus the Logarithm of any given Number may be eaſily found altho' it exceeds the Limits of the Table by 1,203 Places of Figures, and alſo the Number agreeing to any given Logarithm, without the Help of ſuch a Table of proportional Parts, as is uſually inſerted along with the Table of Logarithms for that Purpoſe.

PROB. XIV.

Having given a ſhort Deſcription of Sines, Tangents, &c. we ſhall here ſhew the Geometrical Construction of thoſe and other Scales commonly uſed in projecting the Sphere in Plano and in Trigonometry, Navigation, Dialling, and other Parts of Practical Mathematicks, as they are deduced from a Circle.

Upon a Sheet of fine Paſte-board, or ſuch like Matter, deſcribe a Circle A B D C with any Radius, which croſs at right Angles with the Diameters A B and C D; then continue A D to G, and upon the point B raiſe B F perpendicular to C B. Draw the Chord A B, and divide the Quadrant A B into 9 equal Parts, *Fig.* 6.

Parts, ſetting the Figures 10, 20, 30, &c. to 90 each of which 9 Parts again ſubdivide into 10 equ Parts, and then the Quadrant will be divided int 90 Degrees. Set one Foot of the Compaſſes in th Point A, transfer the ſaid Diviſions to the Chor Line AB, and ſet thereto the Figures 10, 20, 30, &c and the Line of Chords A B will be divided.

To project the *Sines*, divide the Arch BD int 90 Degrees; from each of which Degrees, let fa Perpendiculars on the Semidiameter EB, which Pe pendiculars will divide EB into a Line of Sines, t which ſet the Numbers 10, 20, &c.

To project the *Line of Tangents*, from the Cent E, and thro' every Diviſion of the Arch BD, dra the right Lines cutting BF, which will divide it in to a Line of Tangents, ſet thereto the Numbe 10, 20, &c.

To project the *Line of Secants*, transfer the D ſtances E 10, E 20, &c. From the Tangent Lin upon the Line EG, and ſet thereto the Numbe 10, 20, &c. The Line EG will be divided int Line of Secants.

To project the *Semitangents*, draw Lines from th Point C thro' every Degree of the Quadrant A and they will divide the Semidiameter AE into Line of Semitangents; but becauſe the Semita gents on Scales run to 160 Degrees, continue th Line AE, and draw Lines from the Point C thr the Degrees of the Quadrant CA, cutting AE, an you will have the Line of Semitangents to 160 D grees, &c.

To project the *Rhumb-line*, from every eighth Pa of the Quadrant AC, ſet one Foot of the Compaſſ in A, deſcribe an Arch cutting the Chord AC, whi will divide AC into a Line of whole Rhumbs.

To project the *Line of Longitude*, draw HD equ and parallel to the Radius CE, which divide i to 60 equal Parts, every 10 of which numb Now from every one of theſe Parts let fall Perpen culars to CE, cutting the Arch CD; and havi drawn the Chord CD, with one Foot of the Com paſſes in D, transfer the Diſtances from D to ea

f the Points in the Arch C D on the Chord C D, nd set thereto the Numbers 10, 20, &c. and the Line of Longitude will be divided.

These are all the Lines commonly put upon one Side of the Plain Scale, except equal Parts, which want no Description: And on the other Side is a Decimal or Diagonal Scale, on which an Inch, or some Part thereof, as $\frac{1}{2}$ or $\frac{1}{4}$ is divided into 100 equal Parts, by Diagonals.

Of the Uses of the Chords, Sines and Tangents, &c. *upon the Rule*

The Chords are to lay off the Quantity of an Angle desired upon a given Point in a right Line, and to measure the Quantity of an Angle already laid down. The first is done, by taking the Extent of 60 Degrees of Chords between the Compasses, and describing an Arch about the angular Point; then laying off the Number of Degrees proposed upon the said Arch, and drawing a right Line from the angular Point. And the latter, by making an Arch of 60 Degrees of Chords about the angular Points, and then taking the Chord of the said Arch, between the Compasses, and measuring it on the Line of Chords.

Example, To make an Angle of 30 Degrees on the Point A. Take 60 Degrees of Chords in the Compasses, and setting one Foot in A describe the Arch DC; then take off 30 Degrees from the Chords, lay them off from D to C, and draw the Line A C. The Angle C A B will be 30 Degrees. *Fig.* 7.

To measure an Angle, suppose C A B. Take 60 Degrees of Chords, between the Compasses, and casting one Foot in A, describe the Arch C D; then take the Distance from C to D, which, measured on the Chords, will reach to 30 Degrees, the Quantity of the Angle sought.

The Sines are to project the Sphere orthographically, &c.

The Tangents, Half-Tangents and Secants, are used in finding the Centers and Poles of projected Circles in the Stereographical Projection of the Sphere, &c. The

The Rhumbs, are to lay down the Angles of a Ship's Way in Navigation.

And the Line of Longitude determines, by Inſpection, how many Miles there are in a Degree of Longitude, in each ſeveral Latitude : As, in the Latitude of no Degrees, that is, under the Equator, 60 Miles make a Degree; in the Latitude of 40 Degrees, 46 Miles make a Degree; in the Latitude of 60 Degrees, 30 Miles make a Degree; in the Latitude of 80 Degrees, 10 Miles make a Degree.

Having thus laid the Foundation, we ſhall next ſhew the Reſolution of all right lined Triangles in as plain and familiar Method as poſſible.

SECT. II.

Of reſolving TRIANGLES.

THEOREM I.

Fig. 8. IN any right angled Triangle, if either of the Legs be made Radius, the other Leg will be the Tangent of its oppoſite Angle, *e g.* If A D be made Radius, B D will be Tangent of the Arch d D = B A D; and if B D be made Radius, A D will be Tangent of the Angle B. But if the Hypothenuſe A B be made Radius, the Legs B D and D A will be the Sines of their oppoſite Angles A and B.

THEOREM II.

Fig. 9. *The Sides of every right-lined Triangle are in Proportion to one another, as the Sines of their oppoſite Angle.*

In the Triangle A B C make A F = B C, and let fall the Perpendiculars from F and B to the Side A C, deſcribe the Arches H B and F I. Then B D will be the Sine of the Angle at C, and F E the Sine of the Angle at A, and the Triangles A B D and A F E are ſimilar. Therefore A F : A B : : F E : BD, or as A B : B D : : A F : F E, and A F being equal

to

to B C, and the two Perpendiculars being Sines, it will be, as the Side B C : Side B A : : Sine of the Angle A : Sine of the Angle C; as the Side A B : Sine of the Angle C :: Side B C : Sine of the Angle A.

THEOREM III.

As the Sum of the Legs about an Angle, is to their Difference; so is the Tangent of half the Sum of the opposite Angles, to the Tangent of half their Difference. *Fig. 10.*

Produce, in the Triangle C F D, the Side F D, and make B F = C F; then B D will be the Sum of the Legs, and G D half the Sum; if you take from which the Leg F D, the Remainder G F = $\frac{1}{2}$ the Difference of the Legs; draw C B, and bisect it in A, and draw A F, which will be perpendicular to it, and the Angle C A F = the Angle B F A, (by Prop. 8. *Euclid*, l. 1.) but the Angle C F B = the Angle F C D + the Angle D: (by Prop. 32. *Euclid. l.* 1.) Therefore the Angle C F A = $\frac{1}{2}$ the Sum of the Angles F C D + D; draw A G, which will be parallel to C D, because the Sides C B and B D are bisected in A and G; then draw E F parallel to C D, which will be parallel to A G; the Angle C F E = the Alternate Angle F C D, the lesser Angle of the Triangle, because the Angle C F B = the Angles C + D and E F B = the Angle D, take from both the Angle D, then C F F = the Angle C, which taken from the Angle CFA = $\frac{1}{2}$ the Sum of the opposite Angles, leaves E F A = $\frac{1}{2}$ the Difference of the opposite Angles. Now make A F Radius of a Circle, then E A is the Tangent of half the Difference, and A C the Tangent of half the Sum of the opposite Angles: and the Triangles B A G, B E F and B C D are similar, (by Prop. 2. *Euclid.* l. 6.) and consequently the Sides are proportional. Therefore,

B G : G D :: B A : A C

B G : G F : : B A : AE

Therefore, As G D, half the Sum of the Sides, is to G F half their Difference; so is A C, the Tangent of half the Sum of the opposite Angles, to A E the Tangent of half their Difference.

 But

But the Wholes are as their Halves: Therefore the Sum of the Sides is to their Difference, as the Tangent of half the Sum of the opposite Angles, is to the Tangent of half their Difference.

THEOREM IV.

Fig. 11. *In any Triangle whatsoever, as* ACB, *the Square of the Side* AB *opposite to an acute Angle* C, *is exceeded by the Squares of the other Sides,* AC *and* CB *by the Rectangle* BCF *twice taken; which Rectangle is contained under* BC, *one of the Sides comprehending the acute Angle* C *and the Line* FC, *intercepted between the Perpendicular* AF, *let fall upon the Side* BC *from its opposite Angle* A, *and the acute Angle* C.

The Square of BC = 2 Rectangles BFC and $FC^2 + FB^2$. And $AC^2 = CF^2 + FA^2$ (by Prop. 47. *Euclid. l.* 1) Wherefore, $BC^2 + AC^2 = 2\,BFC + BF^2 + 2\,FC^2 + AF^2$. But $2\,BFC + 2\,FC^2 = 2BCF$. Therefore this being substituted for them; $BC^2 + AC^2 = 2BCF + BF^2 + AF^2$. But $AF^2 + BF^2 = AB^2$ (by Prop. 47. *Euclid. l. 1.*) Therefore this being substituted for them, $BC^2 + AC^2 = 2BCF + AB^2$. That is, $BC^2 + AC^2$ exceed AB^2 by 2BCF.

Fig. 12. The Theorem is true, although the Perpendicular fall without the Triangle. And the Demonstration is almost the same. For $AC^2 = BA^2 + CB^2 + 2CBF$. Add on both Sides, CB^2, then $AC^2 + CB^2 = AB^2 + CB^2 + 2\,CBF = AB^2 + 2\,BCF$.

From this Theorem, and the 47th Prop. **Euclid l. 1** ***we have the Measure of any Triangle whatsoever, whose three Sides are known, although the Area be altogether inaccessible. For by the Help of these Theorems the Perpendicular is known, although the Impediments of the Place should not allow us to mark it out. But note,***

That

That the Perpendicular, multiplied by half the Side on which it falls, produces the Area of the Triangle.

Let there be any Triangle, as ABC, *having its Sides known. It is required, to find the Perpendicular* AF *falling from the Angle* A *upon the opposite Side* BC. *Take the Square of the Side* AB, *opposite to the acute Angle* C, *out of the Sum of the Squares* AC *and* BC: *By the last Theorem, the Remainder shall be the Rectangle* BCF *twice taken. Divide half of the Remainder, that is, the Rectangle* BCF *by the known Side* BC; *thence will arise the right Line* CF. *Take the Square of the Line* CF, *out of the Square of* AC: *The Remainder will give the Square of* AF, *whose square Root will give the Perpendicular* AF. Fig. 11, or 12.

CASE I.

The two acute Angles B *and* C, *and the Base* BA, *being given, to find the Perpendicular* CA. Fig. 12.

1. By making the Hypothenuse BC Radius.

As the Sine of the Angle C at the Perpendicular 56° 15′	9.9198464
Is to the Base AB 121.394	2.0841992
So is the Sine of the Angle B at the Base 33° 45′	9.7447390
	11.8289382
To the Perpendicular AC 81.113	1.9090918

2. By making the Base AB Radius. Fig. 13.

As Radius AB 45°	10.0000000
Is to the Base BA 121.394	2.0841992
So is the Tangent of the Angle B 33° 45′	9.8248926
To the Perpendicular AC 81.113	1.9090918

2. By

Fig. 14. 3. By making the Perpendicular A C Radius.

As the Tangent B A of the Angle C at the Perpendicular 56° 15′	10.1751074
Is to the Radius A C 45°	10 0000000
So is the Base AB 121.394	2.0821992
To the Perpendicular A C 81.113	1.9090918

In making the Proportions for finding the Sides or Angles of a plain Triangle, it must be observed, that every Side of a plain Triangle has two Names, and that each Side has one of those Names fixed, *viz.* the *Hypothenuse*, the *Perpendicular* and *Base*. The other Names are precarious according to the Side made Radius, and are called the Words on the several Sides: Thus, when the Hypothenuse is made Radius, then the Word or the Hypothenuse is Radius, and the Word on the Base is the Sine of its opposite Angle; as also, the Word on the Perpendicular is the Sine of its opposite Angle; but when the Perpendicular is made Radius, then the Word on the Base is the Tangent of its opposite Angle, and the Word on the Hypothenuse is the Secant of the same Angle; and when the Base is made Radius, then the Word on the Perpendicular is the Tangent of its opposite Angle, and the Word on the Hypothenuse is the Secant of that Angle. These Things being observed, the Way to form a Proportion to find the Side of a Triangle, is thus:

First, Suppose one Side of a Triangle to be made Radius, and observe, as above, the Word on the several Sides, the Proportion will be,

As the Word on the Side given,
Is to the given Side;
So is the Word on the Side required,
To the Side required.

Thus we see that what is sought must always stand in the fourth or last Place, and therefore, since the Perpendicular is sought, that must be the last of the four Terms; place it then with the Point of Interrogation after it to shew it is required.

In the Rule of Proportion, the second and fourth Term

Terms being always of the ſame Nature, and the Perpendicular being a Length ſought, and the Baſe the only Length given, the Baſe therefore muſt be in the ſecond Place, and is to be wrote with four Points after it thus : : to ſhew that the Proportion disjoins there.

Again, we are here to obſerve, that the Nature of Logarithms, or their Proportion to one another, is ſuch, that Addition ſerves inſtead of Multiplication, and Subtraction for Diviſion; therefore the Logarithms of the two laſt Terms being added together, and from the Sum, the Logarithm of the firſt Term being ſubtracted, the Remainder 1.9090918 will be the Logarithm of the fourth Term, and the Number anſwering to that Logarithm is 81.113, which is the Perpendicular A C required.

But where Radius is not in Proportion, it may be more readily done by Addition only; for, if inſtead of the firſt Term you ſet its arithmetical Complement, that is, to write down what each Figure wants of 9; thus the arithmetical Complement of 9.9198464, the firſt Term is 0.0801536, which is the ſame as ſubtracting it from 10, then add all the three Terms together, the Sum, abating the Radius, ſhall anſwer the Queſtion

2. To do the ſame by Scale and Compaſſes. Always extend the Compaſſes from the firſt Term to the Term that is of the ſame Kind, whether it be the ſecond or third, that Extent will reach from the remaining Term to the Anſwer. Thus, in the firſt Proportion, extend the Compaſſes from 56°15 to 33°45 in the Line of Sines; that Extent will reach in the Line of Numbers from 121.39 to 81.11 the Anſwer. In the ſecond Proportion, extend the Compaſſes from 45° to 33°45 in the Line of Tangents; that Extent will reach from 121.39 to 81.11 in the Line of Numbers. In ſome Caſes it may be needful to uſe Croſs-work, that is, to extend from the firſt Term in the Line of Sines to the ſecond in the Line of Numbers, or from the firſt Term in Tangents to the ſecond in Numbers, *&c*: but in moſt Caſes it is better to work by the Directions

above, except when the Extent is too large for the Compasses.

3. By the Sliding Rule. Suppose, the Line of Sines on the Rule to be marked with SS, and the Line of Sines on the Slider with S. Then the first Proportion will be thus wrought. Set 33°45′ on S to 56°15′ on SS; then against 121.39 on A, is 81.11 on B. (A signifies the double Line of Numbers upon the Rule, and B the double Number on the Slider). The second Proportion may be thus wrought: Set 33°45′ in the Tangents to Radius; then against 121.39 on A, is 81.11 on B. Or, if the Slider be so turned as the Tangents and double Numbers may slide one by another, then the Radius may be set, *viz.* 45° of Tangents to 121.39 in the Line of Numbers; then against 33°45′ in Tangents is 81.11 in the Line of Numbers. The third Proportion may be wrought as this last.

Fig. 15. 4. To do the same geometrically. Draw the Base BA, and from a Diagonal Scale, or Scale of equal Parts, take with the Compasses 121.39, and set from B to A, and upon A raise a Perpendicular; then take 60 Degrees from the Line of Chords with the Compasses, and set one Foot in B, describe the Arch DE, and from the same Line of Chords 33°45′ and set from D to E; and draw the Line DC till it cut the Perpendicular in C: Then if you measure AC by the same Scale you took BA from, you will find it 81.11; and if you measure BC, you will find it 146. The next Case will be so resolved.

CASE II.

Fig. 12. *The two acute Angles* B *and* C, *and the Base* BA *being given, to find the Hypothenuse* BC.

1. By making the Hypothenuse BC Radius.

As the Sine of the Angle C 56°15′	9.9198464
Is to the Base BA 121.394	2.0841992
So is the Radius 90°	10.0000000
To the Hypothenuse BC 146	2.1643528

2. By

2. By making the Base A B Radius.

s Radius B A	10.0000000	*Fig.* 13.
s to the Base B A 121.394	2.0841992	
o is the Secant of the Angle B 33°45′	10.0801536	
'o the Hypothenuse B C 146	2.1643528	

3. By making the Perpendicular A C Radius.

s the Tangent of the Angle C 56°15′ Arith. Compl.	9.8248926	*Fig.* 14.
s to the Base B A. 121.394	2.0841992	
o is the Secant of the Angle C	10.2552610	
'o the Hypothenuse B C 146	2.1643528	

By Scale and Compasses.

To work the first Proportion extend the Compasses rom the Sine of 56°15 to 90°, which Extent vill reach from 121.39 to 146 in the Line of Numbers.

By the Sliding Rule.

Set 56°15′ upon S to 90° upon S S, then against 21.39 upon B, is 146 upon A. But if you turn the Slider, so as the Sines and double Numbers may slide ne by another, you may set 56° 15′ of Sines to 21. 39 in the Line of Numbers: Then against 90° f Sines you will have 146 in the Line of Numbers, nd will find 33° 45 of Sines to be against 81. 11 of Numbers. So that you may observe, if 90 of Sines e set to the longest Side or Hypothenuse, you will ind every Angle against its opposite Side.

CASE III.

wo acute Angles B *and* C, *and the Hypothenuse* B C *being given, to find the Base* B A. *Fig.* 12.

1. By making the Hypothenuse B C Radius.

As Radius 90°	10.0000000
s to the Hypothenuse B C 146	2.1643528
o is the Sine of the Angle C 56°15′	9.9198464
To the Base B A 121. 394.	2.0841992

2. By

2. By making the Base B A Radius.

Fig. 13. As the Secant of the Angle B 33° 45	10.0801536
Is to the Hypothenuse B C 146	2.1643528
So is the Radius 90°	10.0000000
To the Base B A 121.394	2.0841992

3. By making the Perpendicular A C Radius

Fig. 14. As the Secant of the Angle C 56° 15' Arith Compl.	9.7447390
Is to the Hypothenuse B C 146	2.1643528
So is the Tangent of the Angle C 56° 15'	10.1751074
To the Base B A 121.394	2.0841992

By Scale and Compasses.

To work the first Proportion, extend the Compasses from 90° to 56° 15 in the Sines, which Extent will reach from 146 to 121.394 in the Line of Numbers.

By the Sliding Rule.

Set 56° 15' upon S to 90° upon S S; then against 146 upon A, is 121.394 upon B.

By Geometrical Protraction.

Fig 15. Draw the Line B C, and from the Scale of equal Parts take 146, and set from B to C, with 60 Degrees of Chords describe the Arch D E, and set 56° 15' from E to D, and draw C A, and from B draw BA perpendicular to A C; then when A B is measured the Scale, it will be found to contain 121.39.

CASE IV.

Fig. 13. *The Base* B A *and the Perpendicular* C A *being given, to find the two acute Angles* B *and* C.

1. By making the Base B A Radius.

As the Base B A 121.394	2.0841992
Is to the Radius 45°	10.0000000
So is the Perpendicular A C 81.113	1.9090918
To the Tangent A C of the Angle B 33° 45	9.8248926

Whose Complement is 56° 15' the Angle C.

2. By

2. By making the Perpendicular A C Radius.

As the Perpendicular A C 81.113	1.9090918	*Fig.* 14.
Is to Radius 45° 15'	10.0000000	
So is the Base 121.394	2.0841992	
To the Tangent of the Angle C 56° 15'	10.1751074	

Whose Complement is 33° 45' the Angle B.

By Scale and Compasses.

To work the first Proportion, extend the Compasses from 121.394 to 81.113 in the Line of Numbers, that Extent will reach from 45° to 33° 45' in the Tangents. For the second Proportion, extend from 18 113 to 121.394 in the Line of Numbers, that Extent will reach from 45° to 56° 15' in the Tangents.

By the Sliding Rule.

Let the Tangents and Numbers slide together, then set 121.394 in the Line of Numbers to 45° of Tangents, and against 81.113 in the Line of Numbers is 33° 45' and its Complement 56° 15' in the Line of Tangents.

This Case is done Geometrically, as the sixth.

CASE V.

The Base B A *and the Hypothenuse* B C *being given, to find the two acute Angles* B *and* C. *Fig.* 12.

1. By making the Hypothenuse B C Radius.

As the Hypothenuse B C 146	2.1643528
Is to the Radius 90°	10.0000000
So is the Base B A 121.394	2.0841991
To the Sine of the Angle C 56° 15'	9.9198463

Whose Complement 33° 45 is the Angle B.

2. By making the Base Radius.

As the Base B A 121.394	2.0841992
Is to the Radius 90°	10.0000000
So is Hypothenuse 146	2.1643528
To the Secant of the Angle B 33° 45'	10.0301536

By

By Scale and Compasses.

For the first, extend the Compasses from 146 to 121.394 in the Line of Numbers, which will reach from 90° to 56° 15′ in the Sines.

By the Sliding Rule.

Set 146 upon A to 121.394 upon B, then against Radius on S S, is 56° 15′ on S.

By Geometrical Protraction.

Fig. 15. Draw the Base B A, and from a Scale of equal Parts take 121.394 and set from B to A, and upon A raise the Perpendicular A C; then from the same Scale of equal Parts take 146, and set one Foot of the Compasses in B with the other cross A C in C, and draw B C. Then with 60° of Chords describe the Arch D E, and measure D E with the Compasses on the Line of Chords, which will be 33° 45′ the Measure of the Angle B, whose Complement is the Angle C.

CASE VI.

Fig. 16. *The Base* B A *and the Perpendicular* C A *being given, to find the Hypothenuse* B C

In this, and the next Case, we are first to find the acute Angle, and from thence the third Side.

1. The Perpendicular A C being made Radius.

As the Perpendicular C A 81.113	1.9090918
Is to the Base B A 121.394	2.0841992
So is the Radius 45°	10.0000000
To the Tangent of the Angle C 56° 15′	2.1643528

2. The Hypothenuse B C being made Radius.

As the Sine of the Angle C 56° 15′	9.9198464
Is to the Base B A 121.394	2.0841999
So is the Radius 90°	10.0000000
To the Hypothenuse 146	2.1643528

By

By Scale and Compaſſes.

Extend the Compaſſes from 81.113 to 121 39 in the Line of Numbers, that Extent will reach from 45° to 56° 15′ in the Sine of Tangents; then extend from 56° 15′ to 90° in the Sines, that Extent will reach from 121.394 to 146.

By the Sliding Rule.

Set 81.113 in the Line of Numbers to 45° in the Tangents; then againſt 121.39 in the Numbers is 56° 15′ in the Tangents; then ſet 56° 15′ on S to 90° on SS; then againſt 121.39 on B is 146 on A.

By Geometrical Protraction.

Draw the Line B A, and from a Scale of equal Parts take 121.394 in the Compaſſes, and ſet from B to A, and upon A erect the Perpendicular A C; and from the ſame Scale take 81.113, and ſet from A to C; then draw the Hypothenuſe B C, take B C in the Compaſſes, and on the Scale it will be found 146. Then with 60° of Chords deſcribe the Arch D E, meaſure D E with the Compaſſes on the Line of Chords, which will be 33° 45′ the Meaſure of the Angle B, whoſe Complement is the Angle C. *Fig.* 15.

CASE. VII.

The Baſe B A, *and the Hypothenuſe being given, to find the Perpendicular* A C. *Fig.* 12.

1. The Hypothenuſe B C being made Radius.

As the Hypothenuſe B C 146	2.1643528
Is to the Radius 90°	10.0000000
So is the Baſe B A 121.394	2 0841992
To the Sine of the Angle C 56° 15′	9.9198464
Whoſe Complement is the Angle B 33° 45′	

2. The Hypothenuſe B C being made Radius.

As Radius 90	10.0000000
Is to the Hypothenuſe B C 146	2.1643528
So is the Sine of the Angle B 33° 45′	9.7447390
To the Perpendicular A C 81.113	1.9090918

By

By Scale and Compasses.

For the first Operation, extend the Compasses from 146 to 121.394 in the Line of Numbers, that Extent will reach from 90° to 56° 15′ in the Sines. Then for the second Operation, extend the Compasses from 90° to 33° 45′ in the Sines, that Extent will reach from 146 to 81.113 in the Line of Numbers.

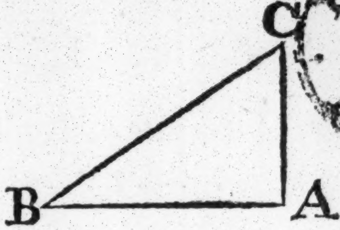
By the Sliding Rule.

For the first, set 146 on A to 121.394 on B, then against 90 on SS there will be found 56° 15′ on S. Then for the second, set 90° on SS to 33° 45′ on S and against 146 on A, is 81.113 on B.

By Geometrical Protraction.

Fig. 15. Draw the Line AB, and from the Scale take 121.394, which set from B to A, and upon A raise the Perpendicular AC, then from the Scale take 146, and set one Foot of the Compasses B, cross the Perpendicular in C, and measure AC on the Scale, which will be 81.113.

The



The Seven Cases of Plain Triangles.

Right Angled.			
Cases.	Given	Required.	Proportions.
I.	A B and B	A C	1. s C : B A : : s B : A C. 2. R : B A : : t B : A C. 3. t C : B A : : R : A C.
II.	A B and C	B C	1. s C : B A : : R : B C 2. R : B A : : seB : B C 3. t C : B A : : seC : B C
III.	B C and B	B A	1. R : B C : : s C : B A 2. seB : B C : : R : B A 3. scC : B C : : t C : B A
IV.	A B and A C	B and C	1. B A : R : : A C : tB whose Complement is C. 2. C A : R : : B A : tC whose Complement is B.
V.	B C and A C	B and C	1. B C : R : : B A : s C whose Complement is B 2. B A : R : : B C : se B whose Complement is C
VI.	A B and A C	B C	1. C A : B A : : R : t C. Then again, 2. s C : B A : : R : B C.
VII.	A B and B C	A C	1. B C : R : : B A : s C whose Complement is B Then again, 2. R : B C : : s B : A C.

SECT. III.

Of solving Oblique Angled TRIANGLES.

CASE I.

Fig. 17. *The Angles* CAB 62° 30′ *and* CBA 37° 30′ *and the Side* AC 350 *Feet being given, to find the other two Sides* CB *and* AB.

As the Sine of the Angle CBA 37° 30′ Arith. Compl.	0.2155529
Is to the Side AC 350	2.5440680
So is the Sine of the Angle CAB 62° 30′	9.9479289
To the Side BC 509.976.	2.7075498

For the Side AB.

As the Sine of the Angle CBA 37° 30′ Arith. Comp.	0.2155529
Is to the Side AC 350	2.5440680
So is the Sine of the Angle ACB 80°	9.9933515
To the Side AB 566.203	2.7529724

The Parts required in the ſeveral Caſes of oblique Trigonometry, may be found with the Scale and Compaſſes by extending the Compaſſes from the firſt Term of the Proportion, to the ſecond, and the ſame Extent will reach from the third Term to the fourth required. Alſo the Parts required, may be found by the Sliding Rule, by ſetting the firſt Term againſt the ſecond, then oppoſite to the third Term, the fourth Term may be found.

CASE II.

Fig. 17. *The two Sides* AC 350 *and* CB 509.976, *and the Angle* CAB 62° 30′ *oppoſite to one of the given Sides,* CB *being given, to find the Angle* CBA *oppoſite to the other Side* AC.

As

As the Side BC 509.976 Arith. Comp.	7.2924502
Is to the Sine of the Angle A 62° 30′	9.9479289
So is the Side A C 350	2.5440680
To the Sine of the Angle B 37° 30′	9.7844471

Or, if the two Sides AB and BC and the Angle opposite to the Side BC had been given, and the Angle C had been required; then,

As the Side B C 509.976 Arith. Comp.	7.2924502
Is to the Sine of the Angle A 62° 30′	9.9479289
So is the Side AB 566.203	2.7529724
To the Sine of the Angle C 80.	9.9933515

CASE III.

The Side AB 566.203, *the Side* AC 350, *and the Angle* A 62° 30 *comprehended between the Sides* AB *and* AC *being given, to find the Angles* ACB *and* ABC. Fig. 17.

AB=566.203		180°	
AC=350	Subt.	62 30′	
The Sum 916.203	Rem.	117 30=	Sum of the Angles B and C.
Diff. 216.203		58 45=	half the Sum.

As 916.203 the Sum of the Sides Arith. Comp.	7.0380083
Is to 216.203 the Difference of the Sides	2.3348617
So is the Tangent 58° 45′ half the Sum of the opposite Angles.	10.2169438
To the Tangent 21° 15′ half the Diff. of the opposite Angles, (by *Theor.* 3. *Sect.* 11.)	9.5898138

If 21° 15′ be added to 58° 45′ the Sum will be 80° the Angle ACB; and if 21° 15′ be subtracted from 58° 45′, the Remainder will be 37° 30′ the Angle ABC.

CASE IV.

Fig. 17. *The Side* A B 566.203, *the Side* B C 509.976 *and the Angle* B 37° 30' *comprehended between the Sides* A B *and* A C *being given, to find the third Side* AC.

The Side A B	566.203	180° 00
The Side B C	509.976	37 30
The Sum	1076.179	142° 30
Difference	56.227	71° 15'

As 1076.179 the Sum of the Sides Arith. Compl.	6.9681154
Is to the Difference 56.227	1.7499449
So is the Tangent of half the Sum of the opposite Angles.	10.4692187
To the Tangent 8° 45' half the Difference of the opposite Angles, (by *Theor*. 3. *Sect*. II.)	9.1871790

If 8° 45' be added to 71° 15', the Sum will be 80° the greater Angle C, and being subtracted the Remainder 62° 30 the lesser Angle A. Then,

As the Sine of the Angle A 62° 30' Arith. Compl.	0.0520711
Is to the Side B C 509.976	2.7075498
So is the Sine of the Angle B 37° 30'	9.7844471
To the Side A C 350, which is required, (by *Case* I. *Sect*. III.)	2.5440680

CASE V.

Fig. 18. *The three Sides* A B 213.5, A C 103.5, *and* B C 250.2 *of an oblique Triangle* A B C *being given to find the three Angles.*

As

As the greateſt Side CB 250.2 Arith. Compl.	7.6017127
Is to the Sum of the other two Sides AB and AC 321.	2.5065050
So is the Difference BE 106 of the two Sides AB and AC.	2.0253059
To the Difference BF 135.995 of the Segments of the Baſe.	2.1335236

Which Difference 135.995, ſubtracted from BC 250.2 the greateſt Side, leaves FC 114.205, the half whereof is GC 57.1025 the leſſer Segment; which if ſubtracted from BC 250.2, the Remainder BG will be 193.0975, the greater Segment of the Baſe. Thus the oblique Triangle is reduced into two right angled Triangles, *viz.* ABG and AGC both right angled at G, in each of which there is given the Hypothenuſe and Baſe: So the Angles may be found by (Caſe V. *Of right angled plain Triangles*;) thus:

As the Hypothenuſe AB 213.5.	2.3293979
Is to the Radius	10.0000000
So is the Baſe BG 193.0975	2.2857762
To the Sine of the Angle BAG 64° 44′ 51″	9.9563783

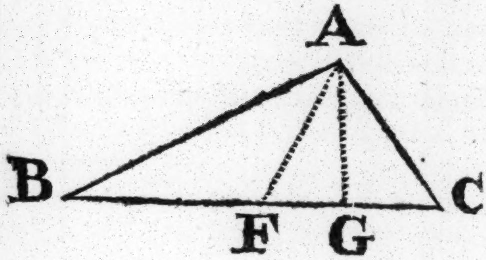
The Complement of BAG is ABG 25° 15′ 9″

Again, in the Triangle ACG.

As the Hypothenuſe AC 107.5	2.0314085
Is to the Radius	10.0000000
So is the Baſe GC 57.1025	1.7566552
To the Sine of the Angle GAC 32° 5′ 8″	9.7252467

The Complement of GAC is ACG 57° 54′ 52″.

This Caſe is demonſtrated from the fourth Theorem of Sect. II.



The Five Cases of oblique angled Triangles.

Cases.	Given.	Required.	Proportions.
I.	A C A and B	A B	s B : A C : : s C : A B.
II.	A C C B and B	A	C A : s B : : C B : s A.
III.	A C C B and C.	A and B	C B + C A : C B − A C : : t $\frac{1}{2}$ A + B : t $\frac{1}{2}$ the Difference, which $\frac{1}{2}$ Dif. { added to / sub. from } the $\frac{1}{2}$ Sum gi. the { greater / lesser } Angle
IV.	A C C B and C.	A B	Find the Angles A and B by the last Case; then, by the first Case, the Side AB will be found.
V.	A B A C & C B	A, B and C	B C : A B + A C : : A B − A C : B F. Then B C − B F = F C and $\frac{1}{2}$ F C is C G.

Sect. I.

Of Trigonometrical Problems.

PROB. I.

To measure an accessible Altitude.

LET AB *represent a Tower, Steeple, &c. whose Height is required?*

First, with the Quadrant, or other Instrument, find the Quantity of the Angle C, which suppose to be 52° 30, then measure the Distance A C; which suppose to be 85 Feet; then by *Case* I. of plain Triangles:

As the Sine of the Angle C B A 37° 30′ Arith. Compl.	0.215553
Is to the Base A C 85 Feet	1.929419
So is the Sine of the Angle C 52° 30′	9 899466
To the Altitude A B 110.8	2.044438

Or thus,

As Radius	10.000000
Is to the Base A C 85	1.929419
So is the Tangent of the Angle C 52° 30′	10.115019
To the Altitude 110.8	2.044438

Note, *That in this, and all such Cases, you must add the Height of your Eye, or Instrument to the Altitude before found.*

PROB. II.

To measure an inaccessible Altitude.

Let AB *be a Church Steeple, whose Height is required; but by reason of a River, or some other Obstacle, you cannot come to the Foot of it at* A.

First, Take, with the Quadrant at C, the Angle of Altitude, which, suppose to be 26° 30; then measure in a right Line towards the Steeple to D, which suppose to be 75 Feet, and at D again observe the Angle of Altitude, which, let be 51° 30.

Now, the two visual Lines C B and D B, and the measured

measured Distance C D form the oblique angled Triangle C B D, wherein are given all the Angles, and the Side C D, the Angle B C D being 26° 30′ and the Complement of A D B 51° 30′ to 180 is the obtuse Angle B D C 128° 30′, and consequently, the third Angle C B D is 25°. But this Angle may be more readily found by subtracting B C D from A D B (by *Euclid.* l. 1. Prop. 32.) Then, by Case I. *Of oblique angled plain Triangles,* find the Side B D; Thus:

As the Sine of the Angle C B D 25° 30′	0.374052
Is to the Distance of C D 75	1.875067
So is the Sine of the Angle C 26° 30′	9.649527
To the visual Line B D 79.18	1.898640

Then by Case I. *Of right angled plain Triangles,*

As Radius	10.000000
Is to B D 79.18	1.989640
So is the Sine of the Angle A D B 51° 32′	9.893544
To the Altitude A B 61.97.	1.729184

PROB. III.

To measure the Height of a Steeple, Tower, &c. standing upon a Hill.

First, Find the Angle C A B 44° and the Angle E A B 26°; then measure in a streight Line towards the Steeple from A to D 134 Feet: Then again at D, find the Angle C D B 67° 50′, and the Angle E D B 51°. By Case I. *Of oblique plain Triangles,* find the visual Line C D in the Triangle A C D, wherein are given the Angles D A C and A C D and the Side A D; Thus:

As the Sine of the Angle A C D 23° 51′	3.393536
Is to the measured Distance A D 134	2.127105
So is the Sine of the Angle C A D 44°	9.841771
To the Side C D 230.4	2.362412

Again,

Then, as Radius	10.000000
Is to the Side C D 230.4	2.362412
So is the Sine of the Angle C D B 67° 50′	9.966653
To the Side B C 213.3	2.329065

Again, As Radius	10.000000
Is to the Side C D 230.4	2.362412
So is the Sine of the Angle B C D 22° 10′	9.576689
To the Base B D 86.92	1.939101

Lastly,

As Radius	10.000000
Is to the Base B D 86.92	1.939101
So is the Tangent of the Angle BDE 51′	10.091631
To the Perpendicular B E 107.3	2.030732

From the whole perpendicular Height BC	213.3
Subtract the perpendicular Height of the Hill B E	107.3
There remains the Height C E of the Steeple	106

PROB. IV.

One Side B C 532 *of an oblique Triangle* A B C, *the Angle* A 110° 30′ *opposite to that Side, and the Sum of the other two Sides* A B *and* A C 637 *being given, to find the other two Sides and the Angles severally.* Fig. 22.

Extend the Side B A to D, make A D equal to A C and draw D C, so there will be other two oblique angled Triangles B D C and A D C. In the Triangle A B C is given the Angle B A C 110° 30′, in the Triangle A C D is given the Angle C A D 69° 30′ the Complement of the other to 180°; also the Triangle A D C is equicrural by Construction, therefore the Angles C and D at the Base are equal, and each of them is equal to ½ the given Angle BAC (by Prop. 32. *Euclid.* l. 1) Now in the Triangle BCD there is given B C 532, B D 637, and the Angle B D C

BDC 55° 15′. Whence the Angle DCB may be found (by Case I. *Of oblique Triangles.*)

As the Side of BC 532 Arith. Comp.	7.2740884
Is to the Sine of the Angle BDC 55° 15′	9.9146852
So is the Side BD 637	2.8041394
To the Sine of the Angle BCD 100° 19′	9.9929130

From which subtract the Angle ACD 55° 15′, and the Remainder is 45° 4′ for the Angle ACB, and the Angle is 24° 26′, which is found by subtracting the Sum of BCD 100° 19′, and D 55° 15′ from 180°. The Sides AB and AC, are found (by Case I. *Of oblique Triangles.*)

As the Sign of the Angle BAC 100° 30′ Arith. Compl.	0.0284124
Is to the Side BC 532	2.7259116
So is the Sine of the Angle ACB 45° 4′	9.8499897
To the Side AB 402.08	2.6043187
Again,	
As the Sine of the Angle BAC 110° 30 Arith Compl.	0.0284124
Is to the Side BC 532	2.7259116
So is the Sine of the Angle ABC 24° 26′	9.6166164
To the Side AC 234.93 ——	2.3709404

PROB. V.

Fig. 23. *One Side* BC 250.2 *of an oblique Triangle* ABC, *the Angle* BAC 96° 50′ *opposite thereto, and the Difference of* BD 106, *of the other two Sides* AB *and* AC *being given, to find the Angles* B *and* C *and the two Sides severally.*

Make AD equal to AC, and draw CD; the Angle DAC being 96° 50′, the Complement thereof to 180° is 83° 10′ for the two Angles ADC and ACD; which being equal one to the other, therefore each of them is half of 83° 10′; and by drawing CD there is also another Triangle made, wherein is

is given BC 250.2 and BD 106, equal to the Difference of the two Sides AB and AC; and there is also given the Angle BDC 138° 25′, equal to the Complement of 41° 35′

As the Side BC 250.2 Arith. Compl.	7.6017127
Is to the Sine of the Angle BDC 138° 25′	9.8219775
So is the Side BD 106	2.0253059
To the Sine of the Angle BCD 16° 19′ 52″	9.4489961

To which if ACD 41° 35′ be added, the Sum of the Angle ACB will be 57° 54′ 52″: And if A be added to it, and the Sum subtracted from 180, there will remain the Angle ABC 25° 15′ 8″. Find the Sides AB and AC, as in the last Problem; Thus:

As the Sine of the Angle BAC 96° 50′ Arith. Compl.	0.0030960
Is to the Side BC 250.2	2.3982873
So is the Sine of the Angle ACB 57° 54′ 52″	9.9280146
To the Side AB 213.5	2.3293979
Again,	
As the Sine of the Angle BAC 96° 50′ Arith. Comp.	0.0030960
Is to the Side BC 250.2	2.3982873
So is the Sine of the Angle ABC 25° 15′ 8″	9.6300247
To the Side AC 107.5	2.0314080

PROB. VI.

In the right angled Triangle ABC *there are given, Base* AB 40, *and the Sum of the Perpendicular* AC, *and Hypothenuse* AD 200, *to find the Perpendicular* AC *and Hypothenuse* CB *severally.* Fig. 24.

In the Triangle ABD, is given the Base and Perpendicular, to find the Angles ABD and ADB; Thus:

As

As the Base AB 40	1.6020600
Is to the Radius 45	10.0000000
So is the Perpendicular AD 200	2.3010300
To the Tangent of the Angle ABD 78° 41′ 24″	10.6989700

Whose Complement is the Angle D 11° 18′ 36″, and because CD is equal to CB, therefore the Angle CBD is also 11° 18′ 36″; then if 11° 18′ 36″ be subtracted from the whole ABD 78° 41′ 24″, there will remain the Angle ABC 67° 22′ 48″. Then to find the Sides.

As Radius	10.0000000
Is to the Base	1.6020600
So is the Tangent of the Angle ABC 67° 22′ 48″	10. 3802083
To the Perpendicular AC 96	1.9822683

Then subtract 96 from 200, and there will remain the Hypothenuse BC 104.

PROB. VII.

Fig. 25. *Let* B, E *and* D *be three Objects, whose Distances are known, and* C *a Station from which all the Objects may be seen, and the Angles with each Object may be found. What is the Distance of each Object?*

BD 106, BE 53, 25 DE 65, 5 *the Angles* BCE 13° 30′ *and* DCE 29° 50′ *are given, to find* BC, EC *and* DC.

The three Angles of the Triangle BDE, *are found by* Case V. *Of oblique angled Triangles.*

Thro' the three Points B, D and C describe a Circle, draw the Lines BC, DC and EC, which last continue to F, where it cuts the Circle, draw BF and DF. Then 180 — Angle BCD = Angle BFD = 136° 40′ (by Prop. 32. *Euclid.* l. 1.) and the Angle BDF = Angle BCF (by Prop. 21. *Euclid.* l. 3.) and so the Angle DBF = Angle DCF. In the Triangle BFD all the Angles are given, and the Side BD, to find BF = 36, 059 to DF = 76, 843

Case I. *Of oblique angled Triangles.*) Then : Angle F D B Angle B D E = F D E; then the Triangle F D E, are given the Sides E D, F D and the included Angle F D E, to find the Angle F E D = 84° 30′ 24″ and 180 — Angle F E D = 95° 29′ 36″; then in the Triangle E D C all the Angles, and the Side D E are given, to find E C = 107, 42 and D C = 131, 05. Lastly, In the Triangle B D C all the Angles and the two Sides B D and D C are given, to find the third Side B C 151.3.

PROB. VIII.

Suppose, B *and* D *two Stations, whose Distance is* 47.5 *Yards, from whence the two Objects* C *and* E, *may may be seen, and their Angles found by Observation,* viz. C B E 49°, E B D 38°, C D B 32°, *and* C D E 56° *it is required to find* B C, B E, D C, D E *and* C E. Fig. 26.

In the Triangle C B D, the Side B D and all the Angles are given, to find B C = 28, 7795, and DC = 54.2349, (which is done by Case I. *Of oblique angled Triangles*) and in the Triangle B E D are given the same Things, to find B E = 58,6775 and DE = 36,1475, (by the same Case.) Lastly, In the Triangle BCE are given B C, B E and the included Angle C B E to find C E = 45,3378, (by Case IV. *Of Oblique angled Triangles.*

PROB. IX.

Let C *and* E *be two Objects, whose Distance is known, and let* B *and* D *represent two Stations, from whence they may be both seen, and the horizontal Angles* C B E *and* D B E, C D E *and* C D B *found by Observation; but the Distance of the two Stations cannot be measured, What is the Distance of the two Stations, and the Objects from each Stations?* Fig. 27.

Draw another Figure c e d b, whose Side d b, suppose to be any Number, as 10, and similar to

CEDB; now, upon Suppoſition, that bd 10, and the ſmall Figure ſimilar to the great one, we can, by the laſt Problem, find de=7,61, be= 12,353116, bc=6,05885, dc=11,41787 and ce=9,5448. Now, if by working in this Manner, ce had been found=CE=45,3378, then it is plain, all the Sides had been exactly found; but as it has not been found by ſimilar Triangles, the true Sides may be found by thoſe already diſcovered, thus, as, ce:CE::cb:CB=28,7795; and as, ce:CE:: cd:CD; and cb:CB::bd:BD=47,5; and as, ce:CE::ed:ED=36,147; and ſo of the reſt.

FINIS.